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- Newton's 2nd Law
  force $F = m \cdot a$ (mass)(acceleration)

  ~~Gen~~ $F = F(t, r, \frac{dr}{dt})$ $\Rightarrow$ for us $F = F(t)$

  ↑time ↑position ↑velocity

Lagrange 1736-1813

Laplace 1749-1827

Legendre 1752-1833

} famous math matician

<u>ex:</u> $F = \langle 6t, 20t^3 - 2, 12t^2 \rangle$

$m = 1$

know $\frac{dv}{dt}$ = acceleration $V = \int a \, dt$

*take anti-derivative $= \langle 3t^2 + c_1, 5t^4 - 2t + c_2, 4t^3 + c_3 \rangle$

$V(0) = \langle 7, 5, -4 \rangle = V(0) = \langle c_1, c_2, c_3 \rangle$

$r(0) = \langle 2, 8, 3 \rangle$

$V(t) = \langle 3t^2 + 7, 5t^4 - 2t + 5, 4t^3 - 4 \rangle$

$r(t) = \int V(t) \, dt$

$= \langle t^3 + 7t + c_1, t^5 - t^2 + 5t + c_2, t^4 - 4t + c_3 \rangle$

$\langle 2, 8, 3 \rangle = r(0) = \langle c_1, c_2, c_3 \rangle$

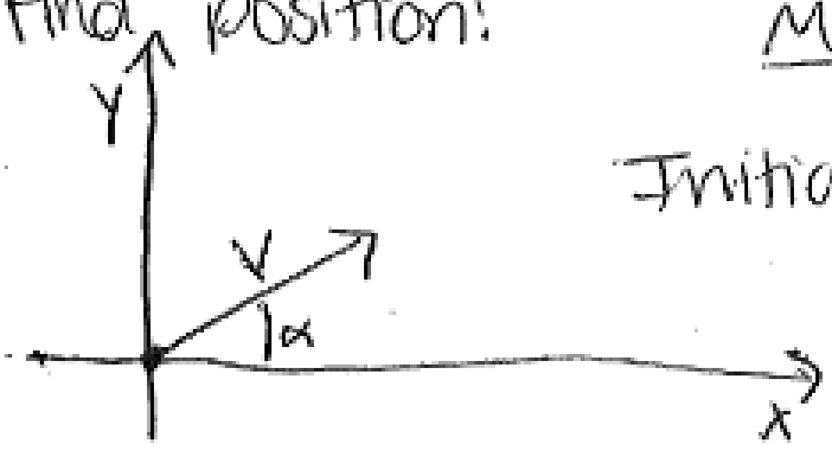
$r(t) = \langle t^3 + 7t + 2, t^5 - t^2 + 5t + 8, t^4 - 4t + 3 \rangle$

- Earth's gravitational force
  $a(t) = \langle 0, 0, -g \rangle$

<u>English</u> $g = 32$ (ft/s²) $\quad v(0) = \langle 2, 1, 5 \rangle$
$\quad r(0) = \langle 1, -7, 9 \rangle$

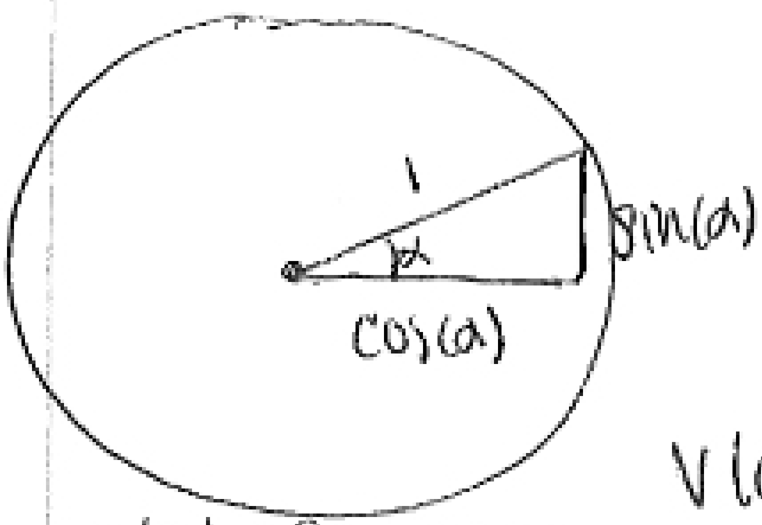
$v(t) = \int a(t)\,dt$
$\quad = \langle c_1, c_2, -32t + c_3 \rangle = v(0) = \langle 2, 1, 5 \rangle$
$v(t) = \langle 2, 1, 5 - 32t \rangle$
$r(t) = \int v(t)\,dt$
$\quad = \langle 2t + c_1, t + c_2, 5t - 16t^2 + c_3 \rangle = r(0) = \langle 1, -7, 9 \rangle$
$r(t) = \langle 2t + 1, t - 7, 5t - 16t^2 + 9 \rangle$

- Find position:



<u>Metric</u> (m/s²)
$a = \langle 0, -9.8 \rangle$
Initial speed $\Rightarrow |v(0)| = 15$
$\alpha = \pi/3$

$\cancel{v(t) = \int a(t)\,dt}$



$\langle \cos(\alpha), \sin(\alpha) \rangle$ the "direction"

$\cos(\pi/3) = 1/2$
$\sin(\pi/3) = \sqrt{3}/2$

$v(0) = 15 \langle 1/2, \frac{\sqrt{3}}{2} \rangle \quad r(0) = \langle 0, 0 \rangle$

$v(t) = \int a(t)\,dt$
$\quad = \langle c_1, -9.81t + c_2 \rangle = v(0) = \langle \frac{15}{2}, \frac{15\sqrt{3}}{2} \rangle$
$v(t) = \langle \frac{15}{2}, \frac{15\sqrt{3}}{2} - 9.81t \rangle \quad r(t) = \int v(t)\,dt$
$\quad = \langle 15/2\, t + c_1, \frac{15\sqrt{3}}{2} t - 4.9t^2 + c_2 \rangle = r(0) = \langle 0, 0 \rangle$

$r(t) = \langle 15/2\, t, 15\sqrt{3}/2\, t - 4.9t^2 \rangle = \langle |v(0)| \cos(\alpha) t, |v(0)| \sin\alpha t - 4.9t^2 \rangle$

Questions that could be asked:

1) What is the maximum height?
2) What is the maximum range?

1) You find maximum height when the y component of the problems velocity equals zero.

ex: $v(t)_y = \frac{15\sqrt{3}}{2} - 9.8t = 0$ $\quad t = \frac{15\sqrt{3}}{2} \cdot \frac{1}{9.81} = \frac{|v(0)|\sin(\alpha)}{g}$

plug into position equation

$$\frac{15\sqrt{3}}{2}\left(\frac{15\sqrt{3}}{2} \cdot \frac{1}{9.8}\right) - \frac{9.8}{2}\left(\frac{15\sqrt{3}}{2} \cdot \frac{1}{9.8}\right)^2$$

simplify

$$= \boxed{\frac{1}{2}(15)^2\left(\frac{\sqrt{3}}{2}\right)^2 \frac{1}{9.8}}$$ ← max. height

2) You find range by having the y-component of the position function equal zero

$\frac{15\sqrt{3}}{2}t_1 - 4.9t_1^2 = 0$ $\quad \left(\frac{15\sqrt{3}}{2} - 4.9t_1\right)t_1 = 0$

$\frac{15\sqrt{3}}{2} - 4.9t_1 = 0$ $\quad \frac{15\sqrt{3}}{2 \cdot 4.9} = 0$ $\quad \boxed{\frac{15\sqrt{3}}{2} \cdot \frac{2}{9.8} = t_1}$

plug into x-component of position function

$$\boxed{\frac{15}{2}\left(\frac{15\sqrt{3}}{2} \cdot \frac{2}{9.8}\right)}$$ ← range