10/28/14

Next Exam Monday 11/10

Physics

Object consisting of a bunch (~10^{23}) of particles
each particle undergoes a force $\vec{F}_i$
m_i position r_i

$$\vec{F}_i = m_i \frac{d^2}{dt^2}(\vec{r}) = \frac{d^2}{dt^2}(m_i\vec{r}_i)$$

external force $\vec{F} = \sum_i \vec{F}_i = \sum_i \frac{d^2}{dt^2}(m_i\vec{r}_i)$
of object

Would like to be able to write $\vec{F} = m\frac{d^2}{dt^2}(\vec{R}) = \frac{d^2}{dt^2}(m\vec{R})$
m total mass $(m = \sum m_i)$
$\vec{R}$ is some fictional position vector.

$$\frac{d^2}{dt^2}(M\vec{R}) = \sum_i \frac{d^2}{dt^2}(m_i\vec{r}_i) = \frac{d^2}{dt^2}\left(\sum m_i\vec{r}_i\right) = \vec{F}$$

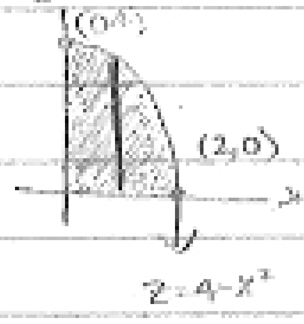
So $M\vec{R} = \sum m_i\vec{r}_i$

$$\vec{R} = \frac{\sum m_i\vec{r}_i}{m} = \frac{\sum m_i\vec{r}_i}{\sum m_i} \qquad m_i = dm$$

Solid in 1^{st} octant bounded by planes $y=0$ $z=0$
and surfaces $z=4-x^2$ & $x=y^2$ with
density function $\delta(xyz) = xy$

tiny volume dV
tiny mass $dm = \delta(xyz)dV$

$$M = \iiint_V \delta \, dV$$



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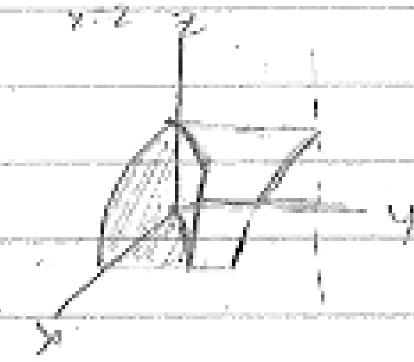

$M = \iiint^{4-x^2} xy\,dz$

$$\boxed{M_x = \int_0^{\sqrt{2}} \int_{y^2}^{2} \int_0^{4-x^2} xy\,dz\,dx\,dy}$$

Solve it!

$= \int_0^{\sqrt{2}} \int_{y^2}^{2} xyz \Big|_0^{4-x^2} dx\,dy$

$= \int_0^{\sqrt{2}} \int_{y^2}^{2} xy4 - x^3y - 0\,dx\,dy$

$= \int_0^{\sqrt{2}} x^2y(2) - \frac{1}{4}x^4y \Big|_{y^2}^{2} dy$

$= \int_0^{\sqrt{2}} 8y - 4y - (2y^5 - \frac{1}{4}y^9)\,dy$

$= \int_0^{\sqrt{2}} 4y - 2y^5 + \frac{1}{4}y^9\,dy$

$= 2y^2 - \frac{1}{3}y^6 + \frac{1}{40}y^{10} \Big|_0^{\sqrt{2}} = 4 - \frac{8}{3} + \frac{4}{5} =$

$\vec{R} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$

$\sum \langle m_i x_i, m_i y_i, m_i z_i \rangle$

$\langle \sum m_i x_i, \sum m_i y_i, \sum m_i z_i \rangle$

X coordinate of center of mass is $\frac{\iiint_V x\,\delta\,dV}{m}$

$\iiint_V x\,\delta\,dV \leftarrow$ 1st moment about x-axis.

So then make inside x^2y & integrate everything again.

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$$\int_0^{\sqrt{2}}\int_{y^2}^{2}\int_0^{4-x^2} x^2y \, dz\, dx\, dy$$

$$= \iint x^2yz \Big|_0^{4-x^2} dx\, dy$$

$$= \iint x^2y4 - x^4y - 0 \, dx\, dy$$

$$= \int \frac{4}{3}x^3y - \frac{1}{5}x^5y \Big|_{x=y^2}^{2} dy$$

$$= \int \frac{32}{3}y - \frac{32}{5}y - \frac{4}{3}y^7 + \frac{1}{5}y^{11} \, dy$$

$$= \frac{32}{15}y^2 - \frac{4}{3\cdot 8}y^8 + \frac{1}{5\cdot 12}y^{12} \Big|_0^{\sqrt{2}} = \frac{64}{15} - \frac{8}{3} + \frac{16}{15}$$

So $\quad \bar{x} = \frac{m_x}{m} = \dfrac{\frac{80}{15} - \frac{40}{15}}{\frac{60}{15} - \frac{40}{15} + \frac{12}{15}} = \boxed{5/4}$

Do same for y & z. Plug in y to integral & solve. Plug in z & solve. etc.

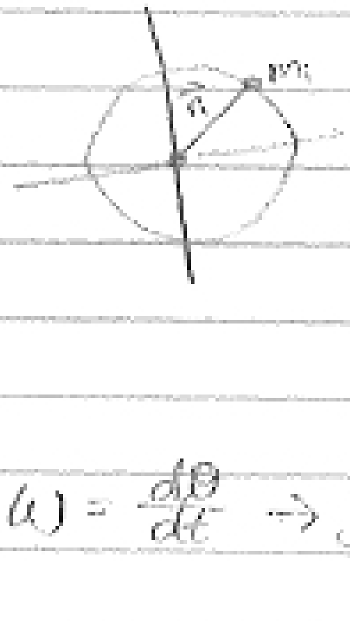
<u>Kinetic Energy</u> $= \frac{1}{2}mv^2 = \sum \frac{1}{2} m_i v_i^2 = \sum \frac{1}{2} m_i \left(r_i \frac{d\theta}{dt}\right)^2$

$$= \sum \frac{1}{2} m_i r_i^2 \omega^2$$

$$= \frac{1}{2}\underbrace{\left(\sum m_i r_i^2\right)}_{\text{mass}}\underbrace{\omega^2}_{\text{velocity}}$$

moment of inertia

$\frac{d}{dt}(\vec{r}_i) = \vec{v}_i \qquad v_i = |\vec{v}_i|$



$\vec{r}_i = \langle r_i \cos\theta, r_i \sin\theta \rangle$

$\vec{v}_i = \frac{d}{dt} r_i$

$= \langle r_i(-\sin\theta)\frac{d\theta}{dt}, r_i(\cos\theta)\frac{d\theta}{dt} \rangle$

$= \langle -\sin\theta, \cos\theta \rangle r_i \frac{d\theta}{dt}$

$v_i = |v_i| = r_i \frac{d\theta}{dt}$

$\omega = \frac{d\theta}{dt} \rightarrow$ same for all particles in object.