engineer notation:

$$\begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix} \bullet \begin{bmatrix} .5 \\ -.2 \\ .7 \end{bmatrix} \quad \text{dot product!}$$

OR

$$\langle 4, 4, 1 \rangle \bullet \langle .5, -.2, .7 \rangle$$

$$= |\langle 4,4,1 \rangle| \, |\langle .5, -.2, .7 \rangle| \cos \theta$$

where 

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$f: \mathbb{R}^n \to \mathbb{R}^1$

nabla $\vec{\nabla} = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle$

if $f: \mathbb{R}^3 \to \mathbb{R}^1$

$$\vec{\nabla} F = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix}$$ gradient of f (grad. f)

$f: \mathbb{R}^n \to \mathbb{R}^1$

$$f(p + \Delta p) \approx f(p) + f'(p)\Delta p$$
$$= f(p) + \nabla f(p) \cdot \Delta p$$

EX) $g: \mathbb{R}^2 \to \mathbb{R}^1$

$g(x,y) = x^2 y + y \qquad g(1,2) = 4$

$p = (1,2)$

Approximation $g(1.02, 1.97) \approx g(1,2) + \nabla g(1,2) \cdot \langle .02, -.03 \rangle$

$\nabla g(x,y) = \langle 2xy, x^2 + 1 \rangle$

$\approx 4 + \langle 4, 2 \rangle \cdot \langle .02, -.03 \rangle = 4 + .08 - .06 = \boxed{4.02}$

$f: \mathbb{R}^1 \longrightarrow \mathbb{R}^1$

$f(x+\Delta x) = f(x) + f'(x)\Delta x + \epsilon_f(x, \Delta x)$

$= f(x) + f'(x)\Delta x + \frac{1}{2}(f''(c)\Delta x^2)$

where c is b/t x & $x+\Delta x$

$f(t) = g(1+(.02)t,\ 2+(-.03)t)$

$f''(t) =$

$g(x+\Delta x, y+\Delta y) \approx g(x,y) + \nabla g(x,y) \cdot \langle \Delta x, \Delta y \rangle$

error $\leq \frac{1}{2} M (|\Delta x| + |\Delta y|)^2$

$M \geq |g_{xx}|, |g_{xy}|, |g_{yy}|$

$g_{xx} = 2y$

$g_{xy} = 2x \qquad 0.98 \leq x \leq 1.02$

$g_{yy} = 0 \qquad 1.97 \leq y \leq 2.03$

$M \geq 4.06, \quad M \geq 2.04$

$\epsilon = \frac{1}{2}(4.06)(|.02| + |.03|)^2$

$\epsilon = 2.03(.0025) = 0.005$

$f(p+\Delta p) \approx f(p) + \nabla f(p) \cdot \Delta p$ $\qquad f: \mathbb{R}^n \to \mathbb{R}$

$\text{error} = \frac{1}{2} m \left(\|\Delta p\|_1 \right)^2$

$m = |f_{xx}|, |f_{xy}|, |f_{yy}|, \ldots$

$\|\Delta p\|_1 = |\Delta x| + |\Delta y| + \ldots$

$g: \mathbb{R}^3 \to \mathbb{R}^1$

$g(x,y,z) = xy + z^2$

$p = (1,2,3)$

$\nabla g(x,y,z) = \langle y, x, 2z \rangle$

$g_{xx} = 0 \quad g_{xy} = 1 \quad g_{yy} = 0 \quad g_{xz} = 0 \quad g_{yz} = 0 \quad g_{zz} = 2$

$\Delta p = \langle .02, -.03, .01 \rangle$

$M \geq 2$

$\text{error} \leq \frac{1}{2}(2)\left((.02) + (.03) + (.01)\right)^2 = \boxed{.0036}$

$\text{approx} = g(1.02, 1.97, 3.01) \approx g(1,2,3) + \nabla g(1,2,3) \cdot \Delta p$

$= 11 + \langle 2, 1, 6 \rangle \cdot \langle .02, -.03, .01 \rangle$

$= \boxed{11.07}$