11/14/14

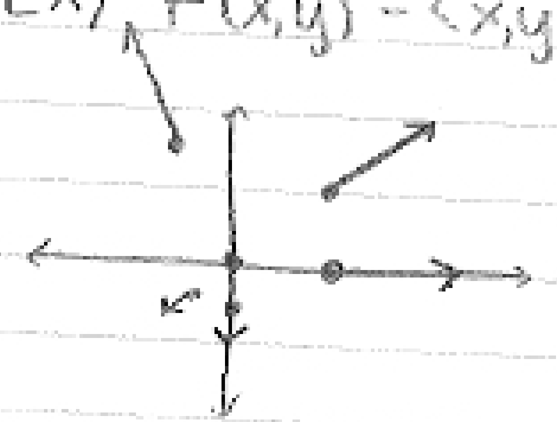
Vector fields and "line" integrals.

curve C: $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ $a \le t \le b$
function: $f(xyz)$

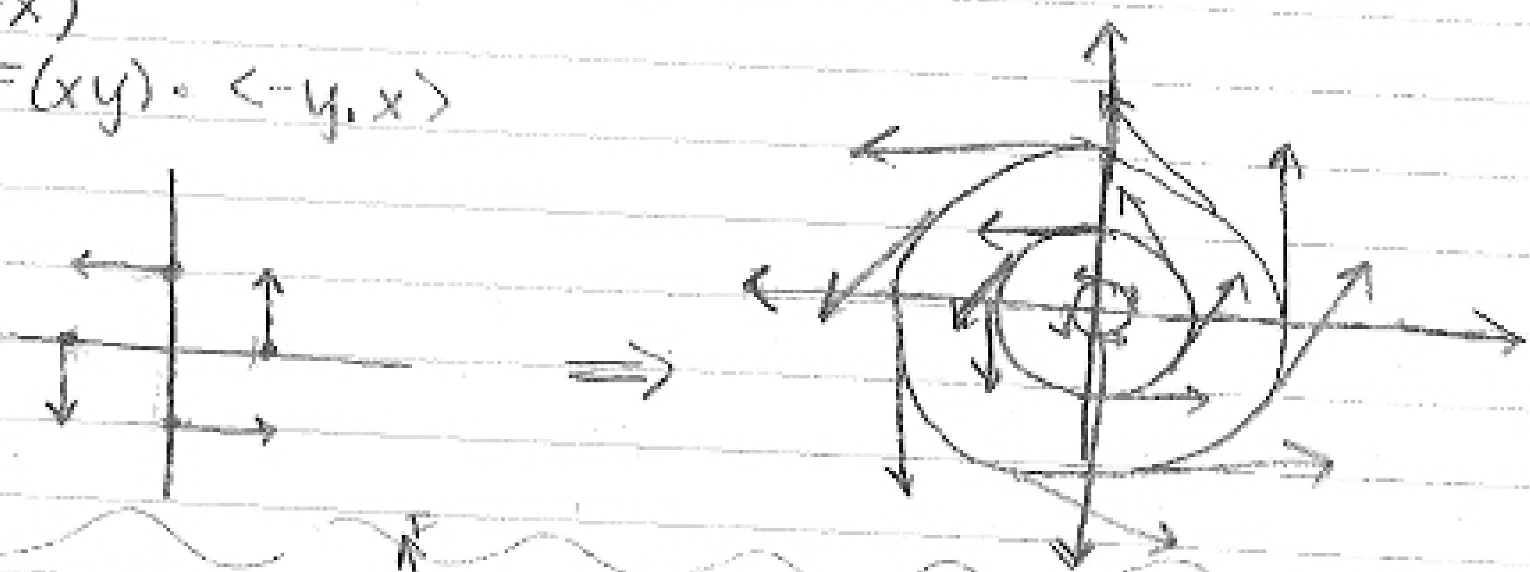
$$\int_C f\,ds = \int_a^b f(x(t), y(t), z(t)) \overbrace{\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}}^{|r'(t)|}\,dt$$

Vector field in $\mathbb{R}^2$
$F: \mathbb{R}^2 \to \mathbb{R}^2$

EX) $F(x,y) = \langle x, y \rangle$



EX)
$F(xy) = \langle -y, x \rangle$



$d\vec{r} = T\,ds$

$\vec{F} = \langle 3, 1, -2 \rangle$
$\vec{D} = \langle -1, 2, 4 \rangle$
$W = -3 + 2 - 8 = -9$
$\boxed{W = -9}$

$W = Fd = F \cdot \text{displacement} = F \cdot D$

$$\int_a^b F\cdot d\vec{r} = \int F\cdot \frac{dr}{dt}\,dt = \int_C \vec{F}\cdot\vec{T}\,ds = \int_a^b \vec{F}\cdot\vec{T}\,ds$$

$$= \int_a^b \vec{F}\cdot\vec{T}\,|r'(t)|\,dt$$

$$T = \frac{r'(t)}{|r'(t)|} \qquad F\cdot T = \frac{F\cdot r'(t)}{|r'(t)|}$$

$$= \int_a^b \frac{F\cdot r'(t)}{|r'(t)|}\,|r'(t)|\,dt = \int_a^b F\cdot r'(t)\,dt$$

$$F = \langle M,N,P\rangle \qquad dr = \langle dx, dy, dz\rangle$$

$$\int_C F\cdot dr = \int_C M\,dx + N\,dy + P\,dz = \int_a^b \left(M\frac{dx}{dt} + N\frac{dy}{dt} + P\frac{dz}{dt}\right)dt$$

EX) $M\,dx + N\,dy + P\,dz = (xy)\,dx + (yz)\,dy + (xz)\,dz$

C_1: $\vec{r}(t) = \langle t,t,t\rangle \quad 0\le t\le 1$ * Goes from

C_2: $\vec{r}(t) = \langle t,t^2,t^4\rangle \quad 0\le t\le 1$ $(0,0,0)\to(1,1,1)$

$C_3 \cup C_4$: C_3: line from $(0,0,0)$ to $(1,1,0)$

C_4: line from $(1,1,0)$ to $(1,1,1)$

C_1: $\int (xy)\,dx + (yz)\,dy + (xz)\,dz$ $x=t \quad \frac{dx}{dt}=1$, $y=t \quad \frac{dy}{dt}=1$, $z=t \quad \frac{dz}{dt}=1$

$$= \int_0^1 t^2\cdot 1 + t^2\cdot 1 + t^2\,dt$$

$$\int_0^1 3t^2\,dt$$

$$t^3\Big|_0^1 = \boxed{1}$$

Not the same

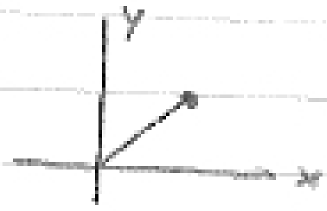
Using other approach for C_2

$C_2 = \int_C (xy)\,dx + (yz)\,dy + (xz)\,dz$ $x=t \quad dx=dt$, $y=t^2 \quad dy=2t\,dt$, $z=t^4 \quad dz=4t^3\,dt$

$$= \int_0^1 t^3\,dt + t^6\,2t\,dt + t^5\,4t^3\,dt$$

$$\int_0^1 t^3\,dt + \int_0^1 2t^7\,dt + \int_0^1 4t^8\,dt$$

$$\tfrac{1}{4}t^4\Big|_0^1 + \tfrac{1}{4}t^8\Big|_0^1 + \tfrac{4}{9}t^9\Big|_0^1 = \tfrac{1}{4} + \tfrac{1}{4} + \tfrac{4}{9} = \boxed{\tfrac{17}{18}}$$

C_3:  $x=t \quad y=t \quad z=0$

$\int_{C_3} (xy)dx + (y-z)dy + (xz)dz$

$x=t \quad dx=dt \qquad = \int_0^1 t^2 dt$

$y=t \quad dy=dt \qquad \frac{1}{3}t^3 \Big|_0^1 = \boxed{\frac{1}{3}}$

$z=0 \quad dz=0$

C_4: $x=1 \quad dx=0 \qquad \int_0^1 t\,dt$

$y=1 \quad dy=0 \qquad \frac{1}{2}t^2 \Big|_0^1 = \boxed{\frac{1}{2}}$

$z=t \quad dz=1$

$C_3 \cup C_4 = \frac{1}{3} + \frac{1}{2} = \boxed{\frac{5}{6}}$ *(Not the same as 1 & $\frac{17}{18}$)

$C_1 C_2 C_3$ & C_4 are the same as B

$M dx + N dy + P dz$: $(y+z)dx + (x+z)dy + (x+y)dz$

C_1: $x=t \quad dx=dt \qquad \int_0^1 2t\,dt + 2t\,dt + 2t\,dt$

$y=t \quad dy=dt \qquad \int_0^1 6t\,dt$

$z=t \quad dz=dt \qquad 3t^2 \Big|_0^1 = \boxed{3}$

C_2: $x=t \quad dx=dt \qquad \int_0^1 (t^2+t^4)dt + (t+t^4)2t\,dt + (t+t^2)4t^3 dt$

$y=t^2 \quad dy=2t\,dt \qquad = \int_0^1 3t^2+5t^4+6t^5\,dt$

$z=t^4 \quad dz=4t^3 dt \qquad t^3+t^5+t^6 \Big|_0^1 = \boxed{3}$

C_3: $x=t \quad dx=dt \qquad \int_0^1 t\,dt + t\,dt$

$y=t \quad dy=dt \qquad = \int_0^1 2t$

$z=0 \quad dz=0 \qquad = t^2 \Big|_0^1 = 1$

C_4: $x=1 \quad dx=0 \qquad \int_0^1 2dt$

$y=1 \quad dy=0 \qquad = 2t \Big|_0^1 = 2 \qquad C_3 \cup C_4 = 1+2 = \boxed{3}$

$z=t \quad dz=dt$