10/3/14

#49 of section 14.6 homework.

$T(x,y) = x(e^{y}+e^{-y})$

$x \approx 2$

$y \approx \ln 2$

max possible errors $|dx| = 0.1$ $|dy| = 0.02$

estimate max possible error in T.

correct value

$T(x+dx,\ y+dy) = T(2+dx,\ \ln 2+dy)$

$T(2+dx, \ln 2+dy) - T(2,\ln 2) = \text{error}.$

$= \cancel{T(2,\ln 2)} + T'(2,\ln 2)\begin{pmatrix} dx \\ dy \end{pmatrix} + \text{error} \cancel{- T(2,\ln 2)}$

$= T'(2,\ln 2)\begin{pmatrix} dx \\ dy \end{pmatrix} + \text{error}$

$= [e^{y}+e^{-y},\ x(e^{y}-e^{-y})]$

$T'(2,\ln 2) = [e^{\ln 2}+e^{-\ln 2},\ 2(e^{\ln 2}-e^{-\ln 2})]$

$= [2+1/2,\ 2(2-1/2)]$

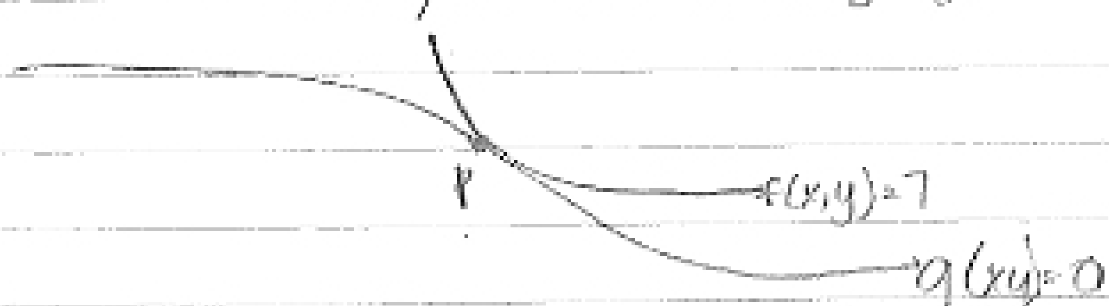
$[2.5,\ 3]$

$[2.5, 3]\begin{bmatrix} dx \\ dy \end{bmatrix}$

$2.5dx + 3dy = 2.5(.1) + 3(.02) = \boxed{.31}$

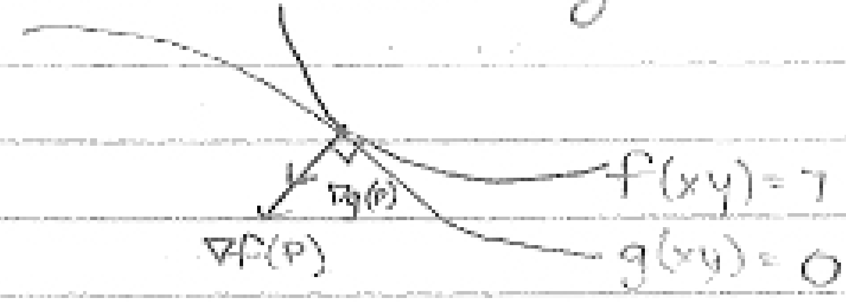
Find extreme values & where they occur of the objective function $f(x,y)$ subject to the constraint $g(x,y) = 0$

Can't have an extreme at p b/c can't cross level curve



Now extreme is possible b/c its tangent

Plan- look for points p on constraint where constraint is tangent to level curve for f.



2 vectors are parallel if you can multiply one by a scaler to equal the other.

So $\nabla f(p) = \lambda \nabla g(p)$ Always put λ by constraint!
$g(p)=1$

EX) $f(xy)=x^2+y^2$ constrain $x^4+2y^4=1$
$g(x,y)=1$

$\nabla f = \langle 2x, 2y \rangle$
$\nabla g = \langle 4x^3 + 8y^3 \rangle$

Quiz question - finding equations

$2x=\lambda 4x^3$ $2y=\lambda 8y^3$ $x^4+2y^4=1$

case $x=0$
$2y^4=1$
$y^4=1/2$
$y=\pm\sqrt[4]{1/2}$

$x \neq 0$
$y=0$
$x^4=1$
$y=\pm 1$

$y \neq 0$
$2=\lambda 4x^2$
$2=\lambda 8y^2$
$\lambda 4x^2 = \lambda 8y^2$
$x^2=2y^2$
$(2y^2)^2+2y^4=1$
$4y^4+2y^4=1$
$y^4=1/6$
$y=\pm\sqrt[4]{1/6}$
$x=\pm\sqrt{2}y$
$x=\pm\sqrt{2}\cdot\sqrt[4]{1/6}$

Points: (8)	$f(x,y)$	Simplified	
$(0, \pm\sqrt[4]{1/2})$	$0+\sqrt{1/2}$	$=\frac{\sqrt{3}}{\sqrt{6}}$	MINS
$(\pm 1, 0)$	1	$=\frac{\sqrt{6}}{\sqrt{6}}$	
$(\pm\sqrt{2}\sqrt[4]{1/6}, \pm\sqrt[4]{1/6})$	$2\sqrt{1/6}+\sqrt{1/6}$	$=\frac{3}{\sqrt{6}}$	MAX

Biggest is max & smallest is min

Find global extremes of $f(xyz) = xyz$
subject to constraint $\underbrace{x^2 + 2y^2 + 3z^2}_{g(xyz)} = 6$

$\nabla f = \langle yz, xz, xy \rangle$ $\qquad$ $\nabla f = \lambda \nabla g$

$\nabla g = \langle 2x, 4y, 6z \rangle$ $\qquad$ $g = 6$

Important part is finding the equations

$yz = \lambda 2x \quad \cdot x = \quad xyz = \lambda 2x^2$

$xz = \lambda 4y \quad \cdot y = \quad xyz = \lambda 4y^2$

$xy = \lambda 6z \quad \cdot z = \quad xyz = \lambda 6z^2$

$x^2 + 2y^2 + 3z^2 = 6$

$\lambda 2x^2 = \lambda 4y^2 = \lambda 6z^2$

$2\lambda(x^2) = 2\lambda(2y^2) = 2\lambda(3z^2)$

$x^2 = 2y^2 = 3z^2$

$x^2 = 2 \qquad x = \pm\sqrt{2}$

$2y^2 = 2 \qquad y = \pm 1$

$3z^2 = 2 \qquad z = \pm\sqrt{2/3}$

Or if λ was zero, then 2

<u>Points:</u> $\qquad$ <u>$f(xyz)$</u>

some pt. $\qquad 0$

$\pm 2/\sqrt{3}$

Global <u>max</u> at 4 points....

Global min at 4 points....