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linear approximations $f(P+\Delta P) \approx f(P) + f'(P)\Delta P$

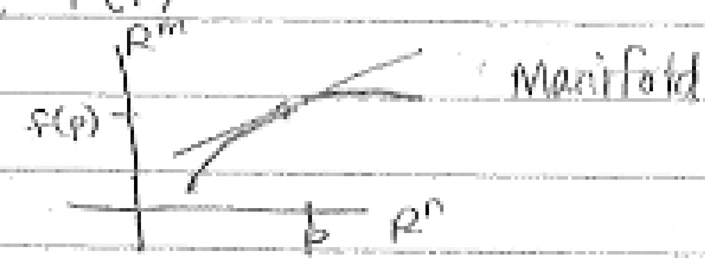
tangent spaces $L(Q) = f(P) + f'(P)(Q-P)$

directional derivatives

$f: \mathbb{R}^n \to \mathbb{R}^m$ $\quad P$ in $\mathbb{R}^n$

Graph of f

$R = f(P)$



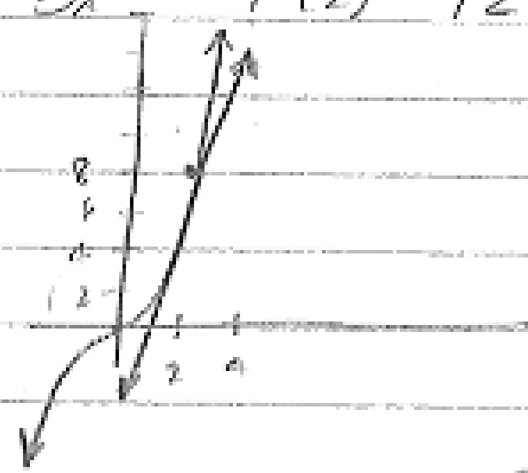
EX) $f: \mathbb{R}^1 \to \mathbb{R}^1$ $\quad P = 2 \quad Q = x \quad f(2) = 2^3 = 8$

$x \mapsto x^3$ $\quad f'(x) = 3x^2 \quad f'(2) = 12$

$L(x) = f(2) + f'(2)(x-2)$

$L(x) = 8 + 12(x-2)$

$y = 12x + 16$



EX) $g: \mathbb{R}^2 \to \mathbb{R}^3$

$g(u,v) = \begin{bmatrix} u^2v \\ uv \\ u-v \end{bmatrix} \quad P = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad g'(u,v) = \begin{bmatrix} 2uv & u^2 \\ v & u \\ 1 & -1 \end{bmatrix}$

$g(1,2) = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$

$g'(1,2) = \begin{bmatrix} 4 & 1 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}$

$L(u,v) = g(1,2) + g'(1,2)\begin{bmatrix} u-1 \\ v-2 \end{bmatrix}$

$= \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} u-1 \\ v-2 \end{bmatrix} = \begin{bmatrix} 4u+v-4 \\ 2u+v-2 \\ u-v \end{bmatrix} = L(u,v)$

Directional Derivatives - All takes place in domain of f : ($\mathbb{R}^n$)

$\mathbb{R}^n$

"if we move away from P, how fast will it increases at time t.

$h(t) = f(P + t\vec{v})$

$h'(0)$ = what we want to know.

$g(t) = P + t\vec{v}$ $\quad g(0) = P$ $\quad g'(t) = \vec{v}$ $\quad g'(0) = \vec{v}$

$h(t) = (f \circ g)(t)$

$h'(t) = f'(g(t))\, g'(t)$

$h'(0) = f'(g(0))\, g'(0)$

$D_{\vec{v}} f(P) = f'(P)\vec{v}$ $\qquad$ directional derivative of f at P in direction of $\vec{v}$

EX) $f : \mathbb{R}^1 \to \mathbb{R}^1$

$f(x) = x^3$

$f'(x) = 3x^2$ $\qquad \mathbb{R}^1$

$P = 2$

$f(2) = 8$

$f'(2) = 12$ $\qquad \vec{v} = [-1]$

$D_{\vec{v}} f(P) = f'(P)\,\vec{v}$

$= f'(2)\,\vec{v}$

$= 12(-1)$

$= [-12]$

$g: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$

$$g(u,v) = \begin{bmatrix} u^2 v \\ uv \\ u-v \end{bmatrix} \quad P = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad g'(u,v) = \begin{bmatrix} 2uv & u^2 \\ v & u \\ 1 & -1 \end{bmatrix}$$

$$\vec{u} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \qquad g'(1,2) = \begin{bmatrix} 4 & 1 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}$$

$$D_{\vec{u}}\, g(1,2) = g'(1,2)\vec{u}$$

$$= \begin{bmatrix} 4 & 1 \\ 2 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \boxed{\begin{bmatrix} 5/\sqrt{2} \\ 3/\sqrt{2} \\ 0 \end{bmatrix} = D_{\vec{u}}\, g(1,2)}$$

$$\vec{v} = \begin{bmatrix} .6 \\ -.8 \end{bmatrix} \qquad D_{\vec{v}}\, g(1,2) = \begin{bmatrix} 4 & 1 \\ 2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} .6 \\ -.8 \end{bmatrix} = \begin{bmatrix} 1.6 \\ .4 \\ 1.4 \end{bmatrix}$$

$f: \mathbb{R}^3 \longrightarrow \mathbb{R}^1 \qquad P \text{ in } \mathbb{R}^n$

$$f'(x,y,z) = [f_x \ \ f_y \ \ f_z] = [y^2 \ \ 2xy \ \ 1]$$

$$f(x,y,z) = xy^2 + z$$

$$f'(1,2,3) = \longrightarrow [4 \ \ 4 \ \ 1]$$

$$P = (1,2,3)$$

$$\Delta P = (.5, -.2, .7)$$

$$f(1.5, 1.8, 3.7) \approx f(1,2,3) + f'(1,2,3)\,\Delta P$$

$$\approx 7 + [4 \ \ 4 \ \ 1] \begin{bmatrix} .5 \\ -.2 \\ .7 \end{bmatrix}$$

$$\approx 7 + (2 - .8 + .7) = \boxed{8.9}$$