$$\left\langle \frac{\partial x}{\partial u} du, \frac{\partial y}{\partial u} du \right\rangle \times \left\langle \frac{\partial x}{\partial v} dv, \frac{\partial y}{\partial v} dv \right\rangle$$

$$= \begin{bmatrix} i & j & k \\ \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & 0 \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & 0 \end{bmatrix} du\,dv$$

$$= \left\langle 0, 0, \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u} \right\rangle du\,dv$$

$$\text{length} = \underbrace{\left| \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial v} \right|}_{\text{Jacobian}} du\,dv = dA$$

$$\iint_A dx\,dy = \iint_B \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$$

11/4/14

Will finish on Tuesday

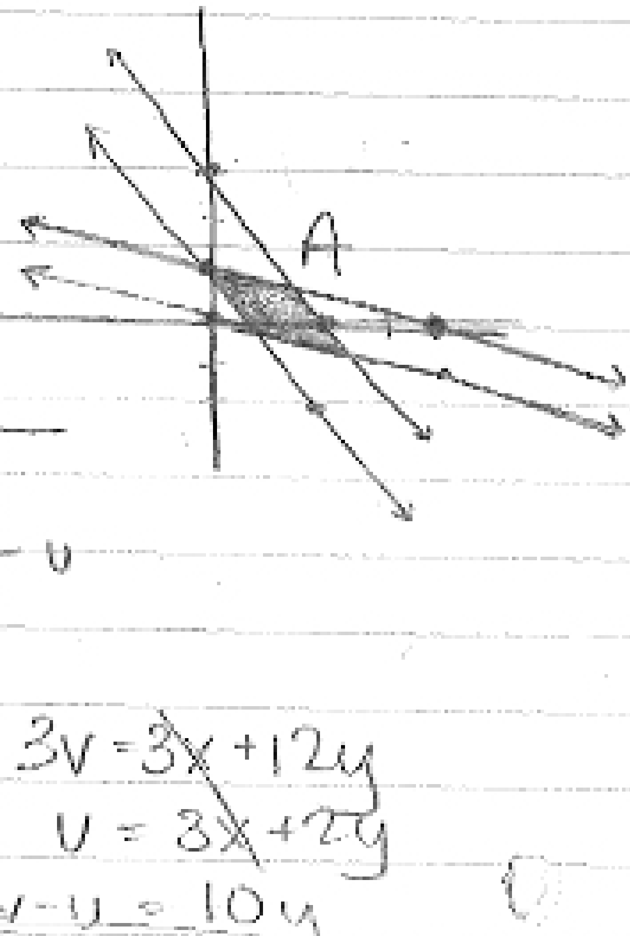
Evaluate

$$\iint_A 3x^2 + 14xy + 8y^2 \, dx\,dy$$

where A is bounded by:

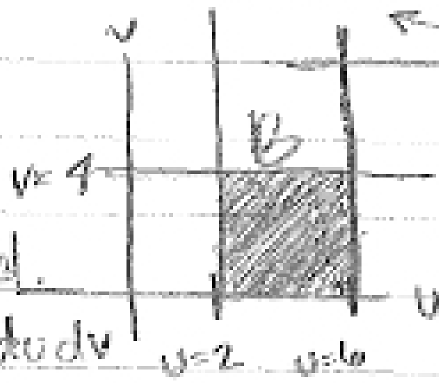
$3x + 2y = 2 \qquad x + 4y = 0$

$3x + 2y = 6 \qquad x + 4y = 4$



let $u = 3x + 2y$

$v = x + 4y$



Now convert integrand.

$$\iint_B 3x^2 + 14xy + 8y^2 \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \det \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix}$$

$$= \det \begin{bmatrix} \frac{2}{5} & -\frac{1}{5} \\ -\frac{1}{10} & \frac{3}{10} \end{bmatrix}$$

$$= \frac{6}{50} - \frac{1}{50} = \boxed{\frac{1}{10}}$$

$3v = \cancel{3x} + 12y$

$u = \cancel{3x} + 2y$

$3v - u = 10y$

$\boxed{-\frac{1}{10}u + \frac{3}{10}v = y}$

$\boxed{\frac{2}{5}u - \frac{1}{5}v = x}$

$$\iint_B 3x^2+14xy+8y^2 \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$$

$$= \int_0^4\int_2^4 3x^2+14xy+8y^2 \left|\frac{1}{10}\right| du\,dv$$

$$\int_0^4\int_2^4 uv\left(\frac{1}{10}\right) du\,dv$$

$$\iint uv\left(\tfrac{1}{10}\right) du\,dv$$

$$\int \tfrac{1}{2}u^2 v \tfrac{1}{10}\Big|_2^4 dv$$

$$\int v\left(\tfrac{9}{5}-\tfrac{1}{5}\right) dv$$

$$= \tfrac{1}{2}v^2 \cdot \tfrac{8}{5}\Big|^4 = \boxed{\tfrac{104}{5}}$$

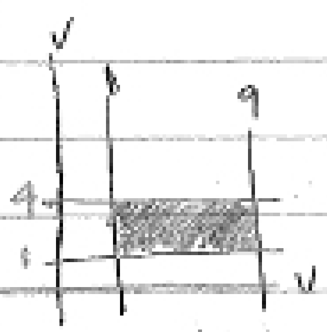
$$\frac{\partial(u,v)}{\partial(x,y)} = \det\begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}\end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 1 & 4\end{pmatrix} \quad 3\cdot4-1\cdot2=10$$

$$\iint_A \sqrt{xy}+\sqrt{y/x}\;dxy$$

A is bounded by

$xy=1$ $\quad y/x=1$

$xy=9$ $\quad y/x=4$

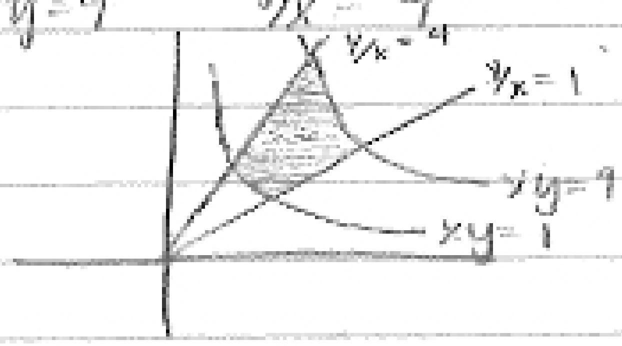


$u=xy$

$v=y/x$

$u=1 \;\&\; 9$

$v=1 \;\&\; 4$



$$\iint_B \sqrt{xy}+\sqrt{y/x}\left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$$

upside down $\rightarrow$ $\frac{\partial(u,v)}{\partial(x,y)} = \begin{pmatrix} y & x \\ -\frac{y}{x^2} & \frac{1}{x}\end{pmatrix} = y\left(\frac{1}{x}\right) - \left(\frac{-y}{x^2}\right)(x) = \frac{y}{x}+\frac{y}{x} = 2\frac{y}{x}$

$$\frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{2\frac{y}{x}} = \boxed{\frac{1}{2}\left(\frac{x}{y}\right)} \text{ — Jacobian}$$

$$\iint \sqrt{u}+\sqrt{v}\;\frac{1}{2}\left(\frac{1}{v}\right) du\,dv$$

$$= \frac{1}{2}\int_1^4\int_1^9 \frac{\sqrt{u}}{v}+\frac{1}{\sqrt{v}}\;du\,dv$$

$$= \frac{1}{2} \int_1^4 \int_1^9 \frac{\sqrt{u}}{v} + \frac{1}{\sqrt{v}} \, du \, dv$$

$$= \frac{1}{2} \int_1^4 \frac{2}{3} \frac{u^{3/2}}{v} + \frac{u}{\sqrt{v}} \Big|_1^9 \, dv$$

$$= \frac{1}{2} \int_1^4 \left(\frac{27 \cdot 2}{3} \cdot \frac{1}{v} + \frac{9}{\sqrt{v}} \right) - \left(\frac{2}{3} \cdot \frac{1}{v} + \frac{1}{\sqrt{v}} \right) dv$$

$$= \frac{1}{2} \int_1^4 \frac{52}{3} \cdot \frac{1}{v} + 8 \frac{1}{\sqrt{v}} \, dv$$

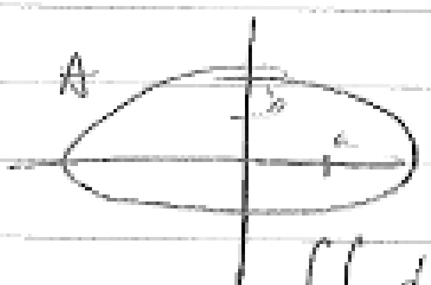
$$\frac{1}{2} \left(\frac{52}{3} \log v + 8 \cdot 2 \sqrt{v} \Big|_1^4 \right) = \frac{1}{2} \left(\frac{52}{3} \log 4 + 32 \right) - \frac{1}{2} \left(\frac{52}{3} \cdot 0 + 8 \cdot 2 \cdot 1 \right)$$

$$= \frac{26}{3} \log 4 + 16 - 8$$

$$\boxed{= \frac{26}{3} \log 4 + 8}$$

Area of Ellipse.

$$\left(\frac{x}{a} \right)^2 + \left(\frac{y}{b} \right)^2 = 1$$



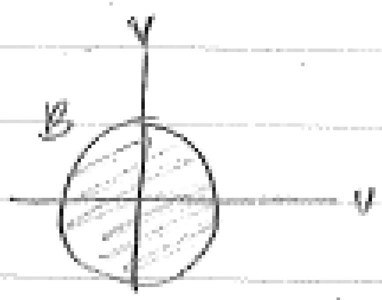
$$\iint_A dA$$

$u = \frac{x}{a}$

$v = \frac{y}{b}$ so $u^2 + v^2 = 1$

$x = au$ $y = bv$ Jacobian $\frac{\partial(x,y)}{\partial(u,v)} = \det \begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix}$

$$= \det \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = ab$$



$$\iint_B ab \, du \, dv$$

$$ab \iint_B du \, dv = ab \pi$$