8/28/14 MATH 2850

13.1 Chain rule

$(f\circ g)' = (f'\circ g)(g')$ $\quad (f\circ g)'(t) = f'(g(t))g'(t)$

$X \xrightarrow{g} Y \xrightarrow{f} Z$ $\quad \mathbb{R} \to \mathbb{R} \to \mathbb{R}^n$

inputs outputs in function of f

$\mathbb{R} \to \mathbb{R}^n$

ex: $g(t) = t^3$ $\quad f(s) = \langle s^4, s^2-3, 7s \rangle$

$(f\circ g)(t) = f(g(t))$

$= \langle g(t)^4, g(t)^2-3, 7g(t) \rangle$

$= \langle (t^3)^4, (t^3)^2-3, 7(t^3) \rangle$

$= \langle t^{12}, t^6-3, 7t^3 \rangle$

$\boxed{(f\circ g)'(t) = \langle 12t^{11}, 6t^5, 21t^2 \rangle}$

or

$g'(t) = 3t^2$ $\quad f'(s) = \langle 4s^3, 2s, 7 \rangle$

$f'(g(t))g'(t) = \langle 4g(t)^3, 2(g(t)), 7 \rangle 3t^2$

$= \langle 4(t^3)^3, 2(t^3), 7 \rangle 3t^2$

$= \langle 4\cdot 3t^9 t^2, 2t^3 3t^2, 21t^2 \rangle$

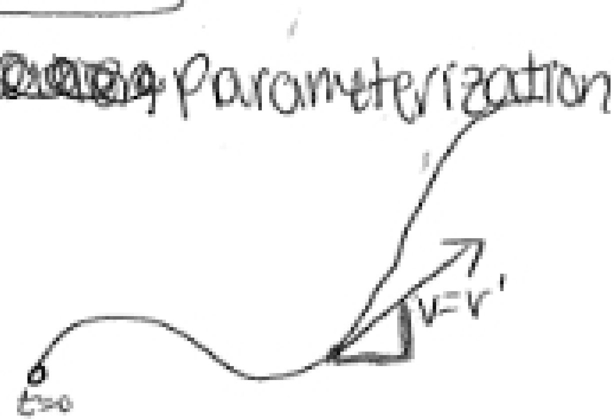
$\boxed{= \langle 12t^{11}, 6t^5, 21t^2 \rangle}$

Promotional message for Parameterization

$r(t) = \langle \frac{t}{t+1}, \frac{1}{t} \rangle$ $\quad t > 0$

$y = f(x)$

$f'(x)$ is slope $= \dfrac{y \text{ comp. of } v}{x \text{ component of } v}$



$x^2+y^2=4$ $\quad r(t)=\langle 2\cos t, 2\sin t\rangle$

$v(t)=r'(t)=\langle -2\sin t, 2\cos t\rangle$

Slope at time t $\quad \frac{2\cos t}{-2\sin t}=-\cot(t)$

$r(t)=\left\langle \frac{t}{t+1}, \frac{1}{t}\right\rangle$ $\quad$ quotient rule: $\frac{gf'-fg'}{g^2}$

$v(t)=r'(t)=\left\langle \frac{(t+1)-t}{(t+1)^2}, -\frac{1}{t^2}\right\rangle$

$=\left\langle \frac{1}{(t+1)^2}, -\frac{1}{t^2}\right\rangle$

Slope at time t:

$$\text{slope}=\frac{\frac{-1}{t^2}}{\frac{1}{(t+1)^2}}=\frac{-(t+1)^2}{t^2}=-\left(1+\frac{1}{t}\right)^2$$

$x=\frac{t}{t+1}$ $\quad y=\frac{1}{t}$ $\quad t=\frac{1}{y}$

$$x=\frac{\frac{1}{\cancel{y}}}{\frac{1}{\cancel{y}}+1}\cdot\frac{y}{\cancel{y}} \Rightarrow x=\frac{1}{1+y}$$

urves in space

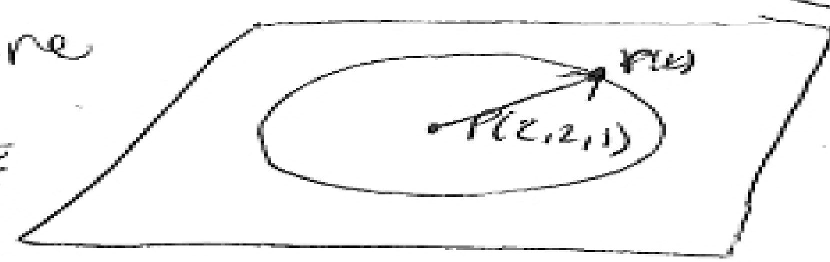
$r(t)=\left\langle 2+\frac{\cos t}{\sqrt{2}}+\frac{\sin(t)}{\sqrt{3}}, 2-\frac{\cos t}{\sqrt{2}}+\frac{\sin t}{\sqrt{3}}, 1+\frac{\sin t}{\sqrt{3}}\right\rangle$

curve lays on this plane.

$x+y-2z=2$

$x+y+z=1$ is a plane

$x^2+y^2+z^2=1$ is a sphere



$r'(t)=\left\langle \frac{-\sin t}{\sqrt{2}}+\frac{\cos t}{\sqrt{3}}, \frac{-\sin t}{\sqrt{2}}+\frac{\cos t}{\sqrt{3}}, \frac{\cos t}{\sqrt{3}}\right\rangle$

(Curves in space continued)

$$r(t)-p=\left\langle \frac{\cos t}{\sqrt{2}}+\frac{\sin t}{\sqrt{3}},\ \frac{-\cos t}{\sqrt{2}}+\frac{\sin t}{\sqrt{3}},\ \frac{\sin t}{\sqrt{3}} \right\rangle$$

$$A+B,\quad -A+B,\quad B$$

$$|r(t)-p|^2=(A^2+2AB+B^2)+(A^2-2AB+B^2)+B^2$$
$$=2A^2+3B^2$$
$$=\frac{2\cos^2 t}{2}+\frac{3\sin^2 t}{3}$$
$$=1$$

$$(r(t)-p)\cdot(r(t)-p)=1$$

$$0=\frac{d}{dt}(1)=r(t)\cdot(r(t)-p)+(r(t)-p)\cdot r'(t)$$
$$=2r'(t)\cdot(r(t)-p)$$

Surface in space

$x^2+y^2+z^2=1$ sphere