9/18/14 MATH 2850

14.3 & 14.4

$\langle 1,2,3\rangle = \begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix}$ $\neq$ *no commas!*

$\begin{bmatrix}1 & 2 & 3\end{bmatrix} = \begin{pmatrix}1 & 2 & 3\end{pmatrix}$

$\begin{bmatrix}1 & 2 & 3\end{bmatrix}\begin{pmatrix}4\\5\\6\end{pmatrix} = (1)(4)+(2)(5)+(3)(6) = 4+10+18 = \underline{32}$

$\neq$

$\begin{bmatrix}4\\5\\6\end{bmatrix}\begin{bmatrix}1 & 2 & 3\end{bmatrix}$ arc $\neq$ car

formula from ch13 from now on written like this $\rightarrow$ $r(t) = \begin{bmatrix}x(t)\\y(t)\\z(t)\end{bmatrix} = \begin{bmatrix}t^3\\3t^2+t\\t^5+8\end{bmatrix}$

- $f(x,y,z) = x^4y^2 + y^4z + xz^3$

The derivative of f $p=(x,y,z)$ $\Delta p = (\Delta x, \Delta y, \Delta z)$

$f(p+\Delta p) = f(p) + f'(p)\Delta p + \cancel{E_f(p)(\Delta p)}$

number

$\begin{bmatrix}f_x(p) & f_y(p) & f_z(p)\end{bmatrix}\begin{bmatrix}\Delta x\\\Delta y\\\Delta z\end{bmatrix} \rightarrow f(p) + f_x(p)\Delta x + f_y(p)\Delta y + f_z(p)\Delta z$

$\begin{bmatrix}\frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z}\end{bmatrix}$ $\leftarrow$ no point of application, Be careful!

$f_x(x,y,z) = 4x^3y^2 + 0 + z^3$

$f_y(x,y,z) = 2x^4y + 4y^3z + 0$

$f_z(x,y,z) = 0 + y^4 + 3xz^2$

$f'(2,1,3) = \begin{bmatrix}4(2)^3(1)^2+(3)^3\\2(2)^4(1)+4(1)^3(3)\\(1)^4+3(2)(3)^2\end{bmatrix} = \begin{bmatrix}59\\44\\55\end{bmatrix}$

$\begin{bmatrix}59 & 44 & 55\end{bmatrix}$ ✓

this needs to be layed down!

~~$f: \mathbb{R} \to \mathbb{R}^3$~~

$f: \mathbb{R}^3 \to \mathbb{R} \to [\quad]$ $\qquad$ $r: \mathbb{R} \to \mathbb{R}^3 \to \begin{bmatrix} \\ \\ \end{bmatrix}$

$\underbrace{f(x,y,z)}_{\text{3 inputs}} \to$ one outputs

input

output $\{ [\quad]$

· input tells you how wide
· output tells you how tall.

Clairut

if f, f_x, f_y
f_{xy}, f_{yz}
all exist and are continuous in neighborhood of P then $f_{xy}(P) = f_{yx}(P)$

ex: $f_x(x,y,z) = 4x^3y^2 + z^3$
$f_y(x,y,z) = 2x^4y + 4y^3z$
$f_{xy}(x,y,z) = 8x^3y + 0$
$f_{yx}(x,y,z) = 8x^3y + 0$

<u>Chain rule:</u>

$(f \circ g)' = (f' \circ g)(g')$
$(f \circ g)'(P) = (f' \circ g)\, g'(P)$
$= f'(g(P))\, g'(P)$

ex: $g(t) = \begin{bmatrix} t^3+t \\ 2t^2+8 \\ t^4 \end{bmatrix}$ $\qquad$ $\mathbb{R}^1 \xrightarrow{g} \mathbb{R}^3 \xrightarrow{f} \mathbb{R}$

$f'(x,y,z) = [f_x(x,y,z) \;\; f_y(x,y,z) \;\; f_z(x,y,z)]$
$= [4x^3y^2+z^3 \;\; 2x^4y+4y^3z \;\; y^4+3xz^2]$

$f \circ g$ $\qquad$ $g'(t) = \begin{bmatrix} 3t^2+1 \\ 4t \\ 4t^3 \end{bmatrix}$

$(f\circ g)'(t) = f'(g(t))\ g'(t)$

$$= \begin{bmatrix} 4(t^3+t)^3(2t^2+8)^2+(t^4)^3 & 2(t^3+t)^4(2t^2+8)+4(t^3+t)^3(t^4) & (2t^2+8)^4+3(t^3+t)(t^4)^2 \end{bmatrix} \begin{bmatrix} 3t^2+1 \\ 4t \\ 4t^3 \end{bmatrix}$$

$$= \left(4(t^3+t)^3(2t^2+8)^2+(t^4)^3\right)(3t^2+1) + \left(2(t^3+t)^4(2t^2+8)+4(t^3+t)^3(t^4)\right)(4t) + \left((2t^2+8)^4+3(t^3+t)(t^4)^2\right)(4t^3)$$

$(f\circ g)'(0) = 0$

$(f\circ g)'(1) = (4\cdot 8\cdot 100+1)4 + 2\cdot 2^4\cdot 10 + \ldots$