9/19/14

$$\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} = (1)(4)+(2)(5)+(3)(6) = 32$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}\begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix} = \begin{bmatrix} (1)(7)+(2)(8)+(3)(9) \\ (4)(7)+(5)(8)+(6)(9) \end{bmatrix} = \begin{bmatrix} 50 \\ 122 \end{bmatrix}$$

$$\begin{tikzcd} \mathbb{R}^2 \arrow[r, "g"] \arrow[rr, bend right, "f \circ g"'] & \mathbb{R}^3 \arrow[r, "f"] & \mathbb{R}^1 \end{tikzcd}$$

$$f(x,y,z) = xy + z^2$$

$$g(u,v) = \begin{bmatrix} u^2 v \\ uv \\ u-v \end{bmatrix} = g\left(\begin{pmatrix} u \\ v \end{pmatrix}\right)$$

$$(f \circ g)'(uv) = (f' \circ g)(u,v)\, g'(u,v)$$

$h: \mathbb{R}^n \to \mathbb{R}^m$

$$\underbrace{\left.\begin{bmatrix} \ \\ h' \\ \ \end{bmatrix}\right\}}_{\text{inputs} = n} \text{outputs} = m$$

$$f'(xyz) = [f_x(x,y,z)\ \ f_y(x,y,z)\ \ f_z(x,y,z)]$$
$$= [y \quad x \quad 2z]$$

$$f'(g(uv)) = [uv,\ u^2 v,\ 2u-2v]$$

$$g'(uv) = \begin{bmatrix} 2uv & u^2 \\ v & u \\ 1 & -1 \end{bmatrix}$$

$$(f \circ g)'(uv) = [uv \quad u^2 v \quad 2u-2v]\begin{bmatrix} 2uv & u^2 \\ v & u \\ 1 & -1 \end{bmatrix}$$

$$= [uv2uv + u^2v^2 + 2u - 2v \qquad u^3 v + u^3 v + 2v - 2v] = \begin{bmatrix} \frac{\partial f \circ g}{\partial u} & \frac{\partial f \circ g}{\partial v} \end{bmatrix}$$

$f \circ g(u,v) = f(g(u,v))$
$= u^3v^2 + (u-v)^2$

This was because $f = xy + z^2$ & $g(u,v) = \begin{bmatrix} u^2v \\ uv \\ u-v \end{bmatrix}$

so product of xy is $(u^2v \cdot uv) = u^3v^2$
and $z^2 = (u-v)^2$ so $[u^3v^2 + (u-v)^2]$

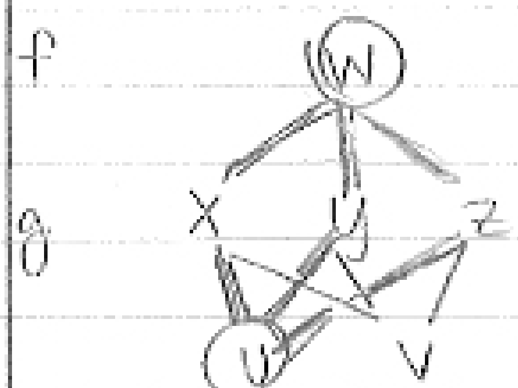
Compute $f \circ g'(1,2) = [3(1)^2(2)^2 + 2(1) - 2(2) \quad 2(1)^3(2) - 2(1) + 2(2)]$
$= [10 \quad 6]$

OR evaluate at point before you multiply

$g(1,2) = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$ $\quad f'(g(1,2)) = f'(2,2,-1)$ *plug this in for $[y \ x \ 2z]$
$= [2, 2, -2]$

$g'(1,2) = \begin{bmatrix} 4 & 1 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}$

$(f \circ g)'(u,v) = [2, 2, -2] \begin{bmatrix} 4 & 1 \\ 2 & 1 \\ 1 & -1 \end{bmatrix} = [2(4) - 2(2) - 2(1) \quad 2(1) + 2(1) + 2(1)]$
$= [10 \quad 6]$



branch diagram *

$$\frac{\partial w}{\partial u} = \left(\frac{\partial w}{\partial x}\frac{\partial x}{\partial u}\right) + \left(\frac{\partial w}{\partial y}\frac{\partial y}{\partial u}\right) + \left(\frac{\partial w}{\partial z}\frac{\partial z}{\partial u}\right)$$

$$(f \circ g)'(u,v) = \begin{bmatrix} \underbrace{3u^2v^2 + 2u - 2v}_{\frac{\partial w}{\partial u}} & \underbrace{2u^3v - 2u + 2v}_{\frac{\partial w}{\partial v}} \end{bmatrix}$$

$$\frac{\partial w}{\partial u} = y\,2uv + xv + 2z(1)$$
$$= (uv)\,2uv + u^2v^2 + 2(u-v)$$
$$= \boxed{3u^2v^2 + 2u - 2v}$$

$x^2 + y^2 + z^2 = 9$ — lets make y the depend variable & x & z are independent.

either $y = \pm\sqrt{9 - x^2 - z^2}$
or use implicit diff.

Diff. with respect to x:

$$2x + 2y\frac{\partial y}{\partial x} + 0 = 0$$

$$2y\frac{\partial y}{\partial x} = -2x$$
$$\frac{\partial y}{\partial x} = \frac{-x}{y}$$

Diff. with respect to z:

$\frac{\partial y}{\partial z} = -z/y$ (used symetry, but works out same way)

$$\boxed{\frac{\partial y}{\partial x} = \frac{-x}{y} \qquad \frac{\partial y}{\partial z} = -z/y}$$