22c:145 Artificial Intelligence

Bayesian Networks

- Reading: Ch 14. Russell & Norvig

Review of Probability Theory

- Random Variables
  - The probability that a random variable X has value val is written as P(X=val)
  - P: domain → [0, 1]
    - Sums to 1 over the domain:
      - » P(Raining = true) = P(Raining) = 0.2
      - » P(Raining = false) = P(¬ Raining) = 0.8
- Joint distribution:
  - **P**(X1, X2, …, Xn)

	Toothache	¬Toothache
Cavity	0.04	0.06
¬Cavity	0.01	0.89

  - Probability assignment to all combinations of values of random variables and provide complete information about the probabilities of its random variables.
  - A JPD table for n random variables, each ranging over k distinct values, has k^n entries!

Review of Probability Theory

- Conditioning
  - $P(A) = P(A \mid B)\ P(B) + P(A \mid \neg B)\ P(\neg B)$
    $= P(A \text{ Æ } B) + P(A \text{ Æ } \neg B)$
- A and B are independent iff
  - $P(A \text{ Æ } B) = P(A) \cdot P(B)$
  - $P(A \mid B) = P(A)$
  - $P(B \mid A) = P(B)$

- A and B are conditionally independent given C iff
  - $P(A \mid B, C) = P(A \mid C)$
  - $P(B \mid A, C) = P(B \mid C)$
  - $P(A \text{ Æ } B \mid C) = P(A \mid C) \cdot P(B \mid C)$

- Bayes' Rule
  - **$P(A \mid B) = P(B \mid A)\ P(A) / P(B)$**
  - **$P(A \mid B, C) = P(B \mid A, C)\ P(A \mid C) / P(B \mid C)$**