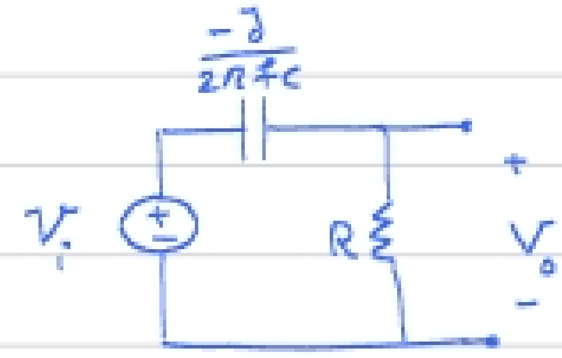
First order RC high pass filters



$$H_{(f)} = \frac{V_o}{V_i} = \frac{R}{R - \frac{j}{2\pi fc}}$$

$$= \frac{1}{1 - \frac{j}{2\pi fRc}} = \frac{2\pi fRc}{2\pi fRc - j1} \times \frac{j}{j} = \frac{j2\pi fRc}{j2\pi fRc + 1}$$

$$\boxed{H_{(f)} = \frac{j\frac{f}{f_B}}{1 + j\frac{f}{f_B}}} \qquad \text{where, } f_B = \frac{1}{2\pi Rc}$$

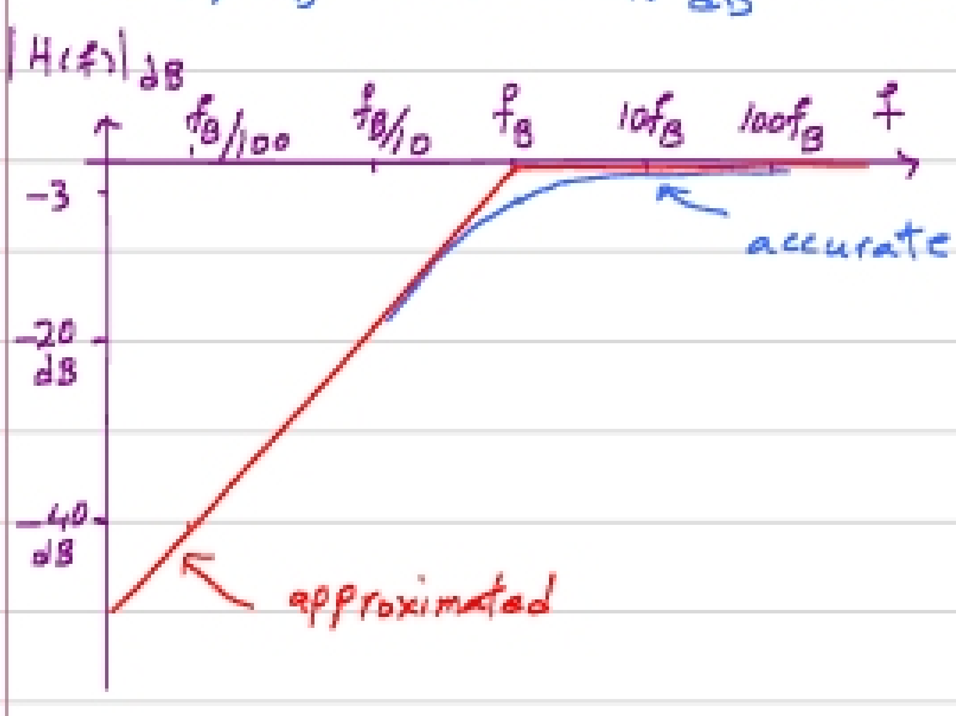
$$\Rightarrow |H_{(f)}| = \frac{\frac{f}{f_B}}{\sqrt{1 + \left(\frac{f}{f_B}\right)^2}} \quad , \quad \angle H_{(f)} = 90 - \tan^{-1}\frac{f}{f_B}$$

straight line with slope of 20 $\frac{dB}{dec}$

If $f \ll f_B \Rightarrow |H_{(f)}|_{dB} = 20\log\frac{f}{f_B}$, $\angle H_{(f)} = 90^\circ$

If $f = f_B \Rightarrow |H_{(f)}|_{dB} = 20\log\frac{1}{\sqrt{2}} = -3\,dB$, $\angle H_{(f)} = 45^\circ$

If $f \gg f_B \Rightarrow |H_{(f)}|_{dB} = 20\log 1 = 0$, $\angle H_{(f)} = 0$



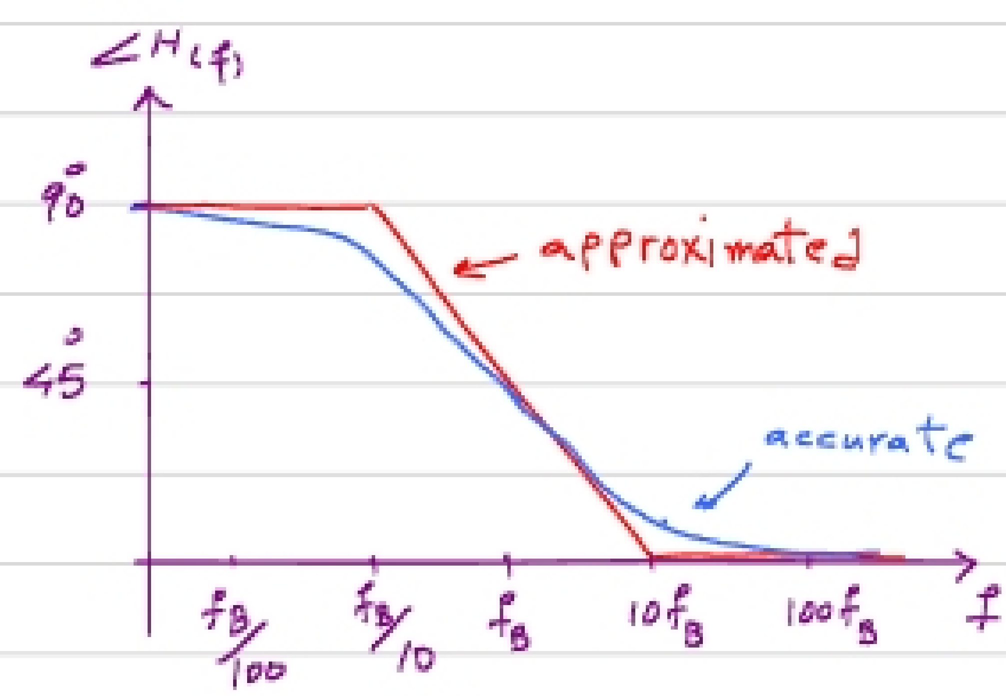


Table 6.2. Transfer-Function Magnitudes and Their Decibel Equivalents

$\lvert H(f)\rvert$	$\lvert H(f)\rvert_{dB}$
100	40
10	20
2	6
$\sqrt{2}$	3
1	0
$1/\sqrt{2}$	−3
1/2	−6
0.1	−20
0.01	−40

$$|H(f)|_{dB} = 20 \log |H(f)|$$

Ex. Find the type of filter. what is the break frequency?

$$H(f) = \frac{V_o}{V_i} = \frac{j2\pi f L}{R + j2\pi f L} = \frac{j2\pi f \frac{L}{R}}{1 + j2\pi f \frac{L}{R}}$$

$$H(f) = \frac{j\frac{f}{f_B}}{1 + j\frac{f}{f_B}} \quad , \quad f_B = \frac{R}{2\pi L}$$

This is the transfer function of a high pass filter with the break frequency of $\frac{R}{2\pi L}$.

It is also possible to find the type of filter using this reasoning:

$Z_L = j2\pi f L \Rightarrow$ at low frequency $(f \approx 0)$ $Z_L \approx 0 \Rightarrow$

inductor is close to a short cct. $\Rightarrow V_o = 0$

$\Rightarrow$ No low frequency components exist at the output.

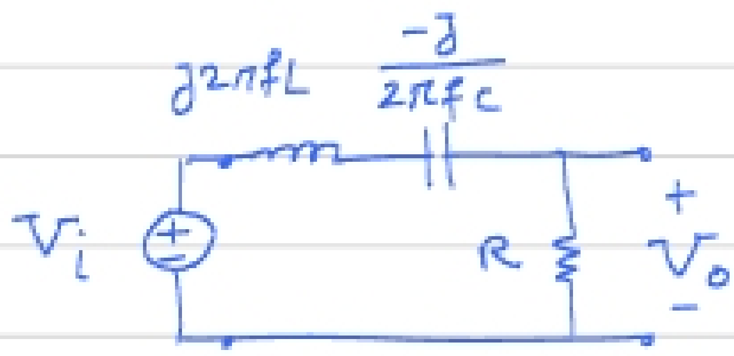
at high freq., Z_L is large (close to open cct.) $\Rightarrow V_o = V_i$

$\Rightarrow$ High frequency component can pass through the filter

Therefore, this is a high pass filter.

Band pass filter

In some applications it is desired to select a certain range of frequencies and discard the other frequencies. Such filters are useful in radio receivers for example.
This filter is called band pass filter and can be made using RLC circuit. (second order circuits)



Total impedance is

$$Z_s = R + j2\pi fL + \frac{-j}{2\pi fC}$$

Resonant frequency is defined as the frequency at which the total impedance is purely resistive. $(Z_L + Z_C = 0)$. Thefore at resonant frequency f_o :

$$j2\pi f_o L - \frac{j}{2\pi f_o C} = 0 \Rightarrow 4\pi^2 f_o^2 LC = 1 \Rightarrow \boxed{f_o = \frac{1}{2\pi\sqrt{LC}}}$$ resonant frequency

Quality factor is defined as:

$$Q_s = \frac{|Z_L|}{R}\bigg|_{f=f_o} = \frac{2\pi f_o L}{R} \quad (1)$$, Since $|Z_L| = |Z_C|$ at $f = f_o$, Q_s is also:

$$Q_s = \frac{|Z_C|}{R}\bigg|_{f=f_o} = \frac{1}{2\pi f_o RC} \quad (2)$$, Then the total impedance can be written as:

multiplied by Q_s from (1)   multiplied by Q_s from (2)

$$Z_s = R\left[1 + jQ_s\left(\frac{f}{f_o} - \frac{f_o}{f}\right)\right]$$

$$H(f) = \frac{V_o}{V_s} = \frac{R}{Z_s} = \frac{1}{1 + jQ_s\left(\frac{f}{f_o} - \frac{f_o}{f}\right)}$$

$$|H(f)| = \frac{1}{\sqrt{1 + Q_s^2\left(\frac{f}{f_o} - \frac{f_o}{f}\right)^2}}$$