Lecture 9

- Agenda:
  - MEMS Inertial Sensors
  - Coriolis Force
  - Principle of Vibratory Gyroscope

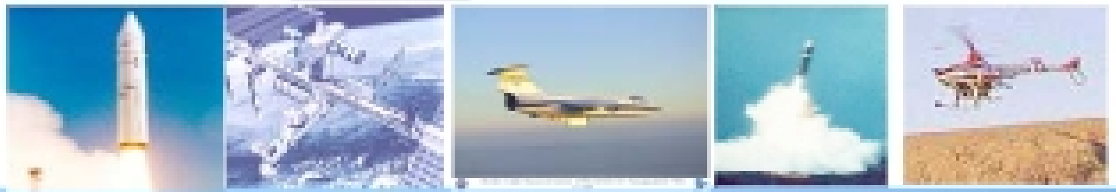
MEMS Inertial Sensors

Emerging applications

- **Low cost** – Automobiles, computer games, motion detection
- **Small size** – Camcorders, nano-satellites, health monitoring
- **Integrated Inertial Measurement Unit (IMU) system**

➡ **MEMS inertial sensors**

Traditional applications of inertial sensors



Inertial Sensors

- **Accelerometers**
  - Single-axis, dual-axis, 3-axis
  - Types
    - Capacitive
    - Piezoresistive
    - Piezoelectric
    - Tunneling
    - Optical

- **Commercial Products:**
  ADI, Motorola, Bosch, Honeywell, ST Microelectronics, ...

Inertial Sensors

Gyroscopes -- Angular rate sensors

·Optical gyroscopes
- Fiber-optic gyroscope
- Ring laser gyroscope

·Mechanical gyroscopes
- Spinning wheel
- Vibrating fork, shell, plate

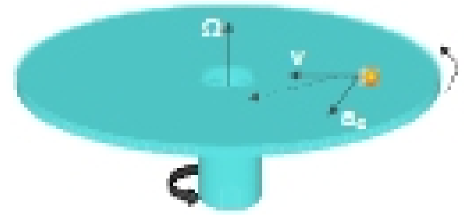
-> Micromachined vibratory gyroscopes

⟹ Coriolis Acceleration

Coriolis Force

A moving ball on a stationary table

A moving ball on a rotating table



A mass-spring system on a rotating table

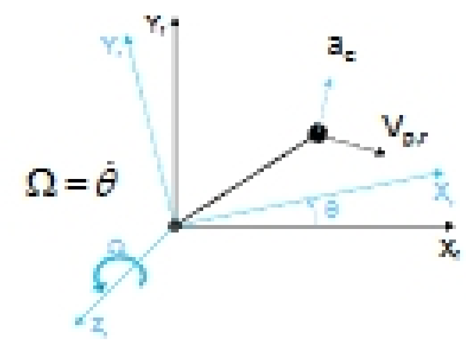
$\vec{a}_c$: Coriolis acceleration

$$\vec{a}_c = 2\vec{\Omega}\times\vec{V}$$

Coriolis force is a fictitious force exerted on a body when it moves in a rotating reference frame

Coriolis Acceleration

A particle in a rotating frame



Rotation	Vibration	Coriolis acceleration
Ω_x	y ↔	z
Ω_y	z ↔	x
Ω_z	x ↔	y

$$\vec{a}_{p,f} = \vec{a}_{p,r} + \vec{a}_{r,f} + \vec{a}_c$$

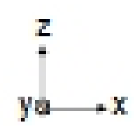
$$\vec{a}_c = 2\cdot\vec{\Omega}\times\vec{V}_{p,r}$$

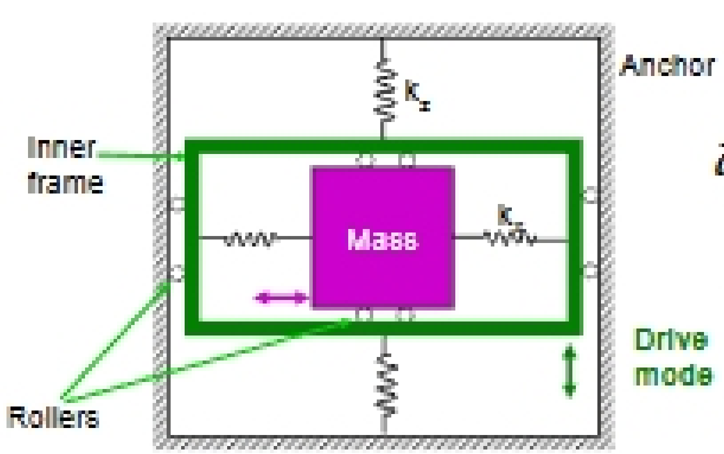
Xr-Yr: Rotating frame
Xf-Yf: Fixed frame

- $\vec{a}_{p,f}$ – Acceleration of the particle w.r.t the fixed frame
- $\vec{a}_{p,r}$ – Acceleration of the particle w.r.t the rotating frame
- $\vec{a}_{r,f}$ – Acceleration of the rotating frame w.r.t. the fixed frame at P
- $\vec{a}_c$ – Coriolis acceleration
- $\vec{V}_{p,r}$ – Velocity of the particle w.r.t the rotating frame

Vibratory Gyroscope - model

Gimbaled Symmetric Design





$$\vec{a}_{c\_z} = 2\vec{\Omega}_x\times\vec{V}_y$$

Electrostatic,
piezoelectric,
electromagnetic,
thermal

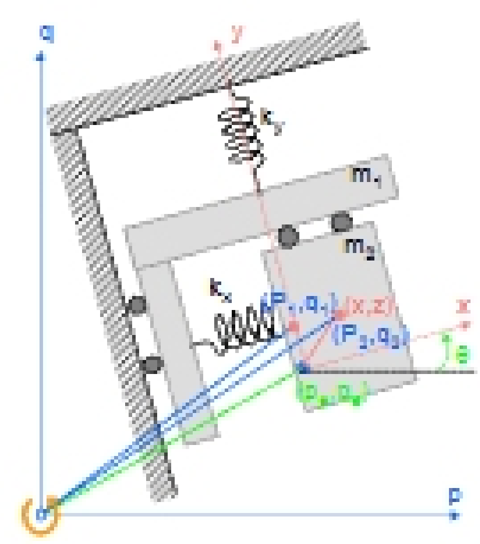
Sense mode

Capacitive
Piezoresistive
Piezoelectric.

Oscillating accelerometer

Vibratory Gyroscope - EOM

Use Lagrange's equation,



p-q: fixed frame
x-z: rotating frame

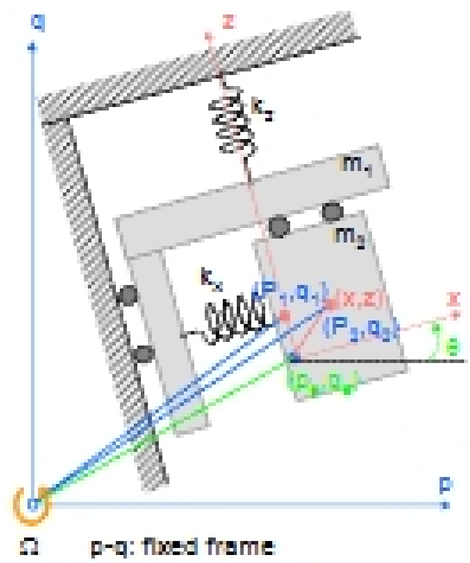
$$\begin{cases} T = \frac{1}{2}m\dot{p}^2 + \frac{1}{2}m\dot{q}^2 + \frac{1}{2}I\dot{\theta}^2 \\ V = \frac{1}{2}k_x x^2 + \frac{1}{2}k_z z^2 \end{cases}$$

where $\begin{cases} p(t) = x\cos\theta - z\sin\theta + p_o \\ q(t) = x\sin\theta + z\cos\theta + q_o \end{cases}$

$$L = T - V$$

$$\begin{cases} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0 \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) - \frac{\partial L}{\partial z} = F_z \end{cases}$$

Vibratory Gyroscope - EOM



p-q: fixed frame
x-z: rotating frame

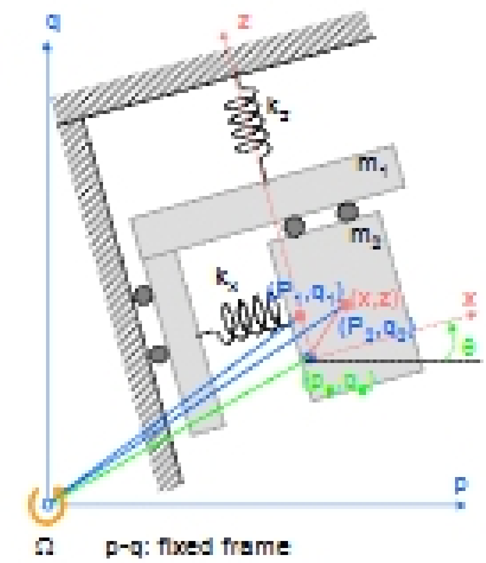
Coriolis acceleration

$$\begin{cases} \ddot{x} + \dfrac{D}{m_2}\dot{x} + \left(\dfrac{k_x}{m_2} - \dot{\theta}^2\right)x - 2\dot{\theta}\dot{z} \\ \quad - \ddot{\theta}z + \ddot{p}_a \sin\theta + \ddot{q}_a \cos\theta = 0 \\ \ddot{z} + \dfrac{D}{m_1+m_2}\dot{z} + \left(\dfrac{k_z}{m_1+m_2} - \dot{\theta}^2\right)z + \dfrac{2m_2}{m_1+m_2}\dot{\theta}\dot{x} \\ \quad + \dfrac{m_2}{m_1+m_2}\ddot{\theta}x + \ddot{p}_a\cos\theta - \ddot{q}_a\sin\theta = F_z \end{cases}$$

Where D -- Damping coefficient

$$\Omega = \dot{\theta}$$

Vibratory Gyroscope - EOM



p-q: fixed frame
x-z: rotating frame

Coriolis acceleration

$$\begin{cases} \ddot{x} + \dfrac{D}{m_2}\dot{x} + \left(\dfrac{k_x}{m_2} - \dot{\theta}^2\right)x - 2\dot{\theta}\dot{z} \\ \quad - \ddot{\theta}z + \ddot{p}_a \sin\theta + \ddot{q}_a \cos\theta = 0 \\ \ddot{z} + \dfrac{D}{m_1+m_2}\dot{z} + \left(\dfrac{k_z}{m_1+m_2} - \dot{\theta}^2\right)z + \dfrac{2m_2}{m_1+m_2}\dot{\theta}\dot{x} \\ \quad + \dfrac{m_2}{m_1+m_2}\ddot{\theta}x + \ddot{p}_a\cos\theta - \ddot{q}_a\sin\theta = F_z \end{cases}$$

Where D -- Damping coefficient

$$\Omega = \dot{\theta}$$

Vibratory Gyroscope

The sensing displacement

$$|x|\angle\varphi_s = K_s\frac{|a_c|}{\omega_{r,s}^2}\angle\varphi_s = K_s\frac{2|v_y||\Omega_z|}{\omega_{r,s}^2}\angle\varphi_s$$

where $K_s = \left[\left(1-\left(\frac{\omega_{r,d}}{\omega_{r,s}}\right)^2\right)^2 + \left(\frac{\omega_{r,d}}{Q_s\omega_{r,s}}\right)^2\right]^{-1/2}$ $\qquad \varphi_s = \tan^{-1}\left[\frac{\omega_{r,d}}{Q_s\omega_{r,s}}\left(1-\left(\frac{\omega_{r,d}}{\omega_{r,s}}\right)^2\right)^{-1}\right]$

Mechanical Sensitivity

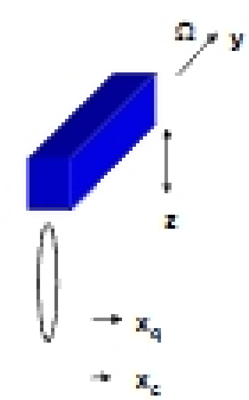
$$\frac{x_{sm}}{y_{dm}}\Big/\Omega_{sm} = 2K_s\frac{\omega_{r,d}}{\omega_{r,s}^2}$$

Brownian Noise

$$\frac{\Omega_{sm}}{\sqrt{BW}} = \sqrt{\frac{k_B T\omega_{r,s}}{m_s Q_s \omega_{r,d}^2 y_{dm}^2}}$$

Quadrature

- Quadrature is the coupled motion due to anisoelasticity of microstructures.
- Drive in z, but x motion may exist.



90° phase difference