AAE 340 PS9 F13 SOLUTION DAVIDE GUZZETTI

PROBLEM 1

Given An aircraft, inertially Fixed unit vectors $\hat{n}$ body-Fixed unit vectors $\hat{s}$

Intermidiate Frames $\hat{a}$ (1st) anb $\hat{b}$ (2nd)

Find (a) Define yaw-pitch-roll as 2-1-3 (η, σ, γ) body sequence

(b) $[\ell]_{\hat{n}\cdot\hat{a}}$, $[\ell]_{\hat{b}\cdot\hat{a}}$, $[\ell]_{\hat{n}\cdot\hat{s}}$, $[\ell]_{\hat{n}\cdot\hat{s}}$

(c) ${}^{n}\bar{\omega}^{s}$ in Terms of The $\hat{s}$-Frame and the $\hat{b}$-Frame

(d) Evaluate $[\ell]_{\hat{n}\cdot\hat{s}}$ for $(10°, -15°, 20°)$ and verify the ortogonality conditions are satisfied.

(e) $\dot{\sigma} = 1$ deg/s $\dot{\eta} = 2$ deg/s $\dot{\gamma} = 0$ deg/s

Evaluate ${}^{b}\bar{\omega}^{s}$ and ${}^{n}\bar{\omega}^{s}$, Also $|{}^{b}\bar{\omega}^{s}|$, $|{}^{n}\bar{\omega}^{s}|$

Solution

(a) $\hat{n}$ initial Frame

$\hat{a}$ First intermediate Frame

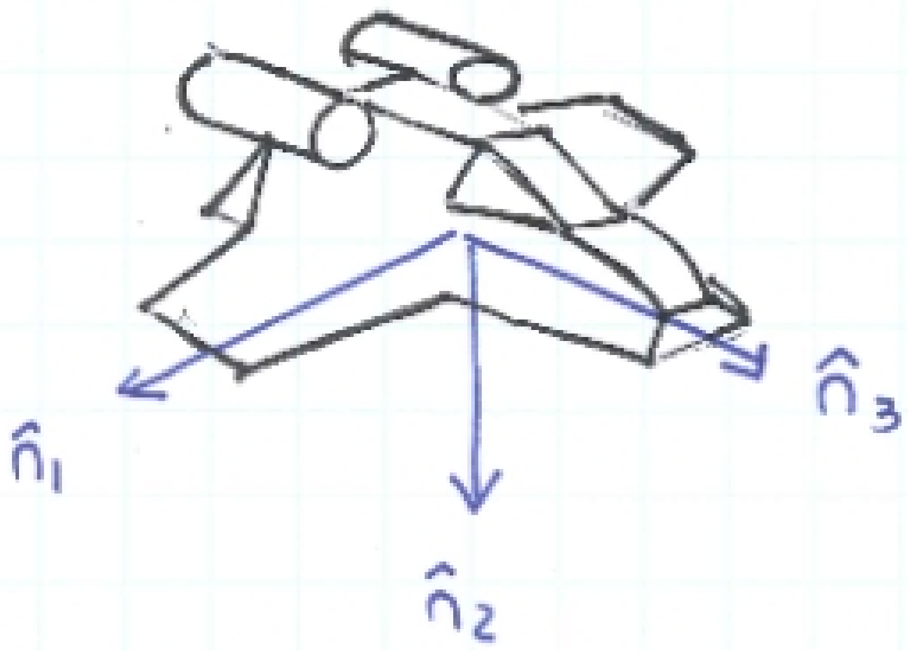
$\hat{b}$ second intermidiate Frame

$\hat{s}$ body Frame

$\hat{n} \to \hat{a}$: ROT #1 η

$\hat{a} \to \hat{b}$: ROT #2 σ

$\hat{b} \to \hat{s}$: ROT #3 γ

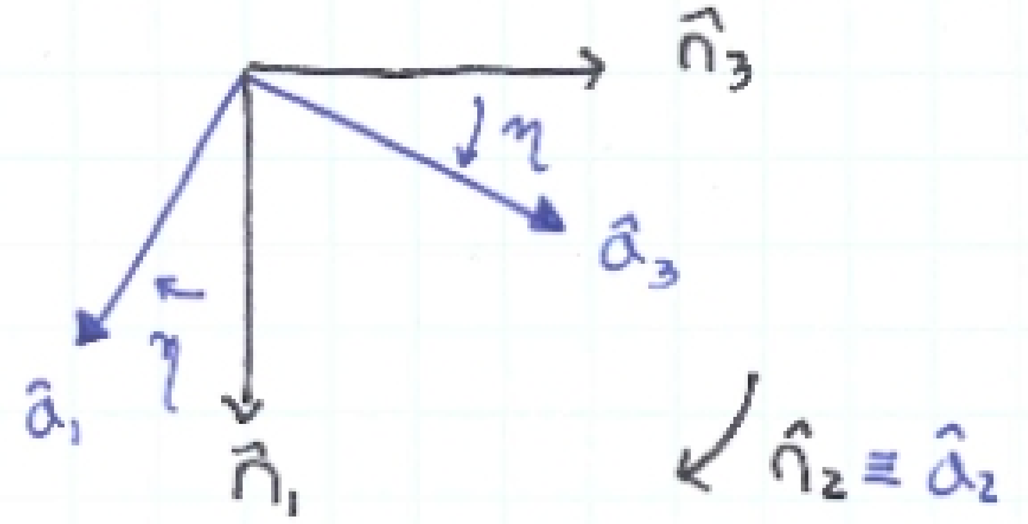


INERTIAL FRAME $\hat{n}$

when ALL the angles (η, σ, γ) are ZERO $(0,0,0)$

$\hat{n}_2$: directed downward
$\hat{n}_3$: along the Fusolage
$\hat{n}_1$: out of the right wing

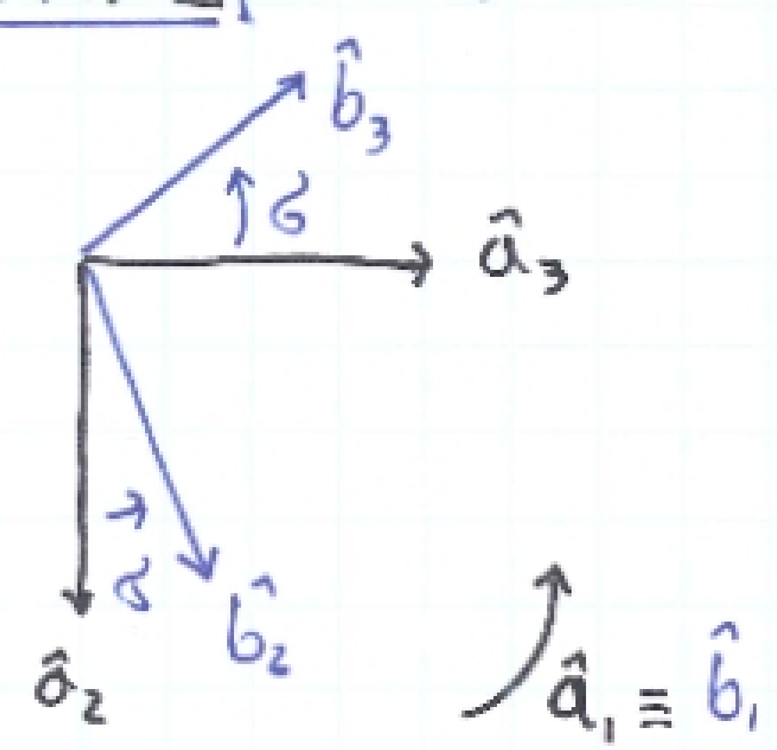
→ **ROT#1** $\quad {}^{\hat{n}}\bar{\omega}^{\hat{a}} = \dot{\eta}\,\hat{n}_2 = \dot{\eta}\,\hat{a}_2$ $\quad$ YAW



	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$
$\hat{n}_1$	c_η	0	s_η
$\hat{n}_2$	0	1	0
$\hat{n}_3$	$-s_\eta$	0	c_η

$= [\ell]$
$\hat{n}\cdot\hat{a}$

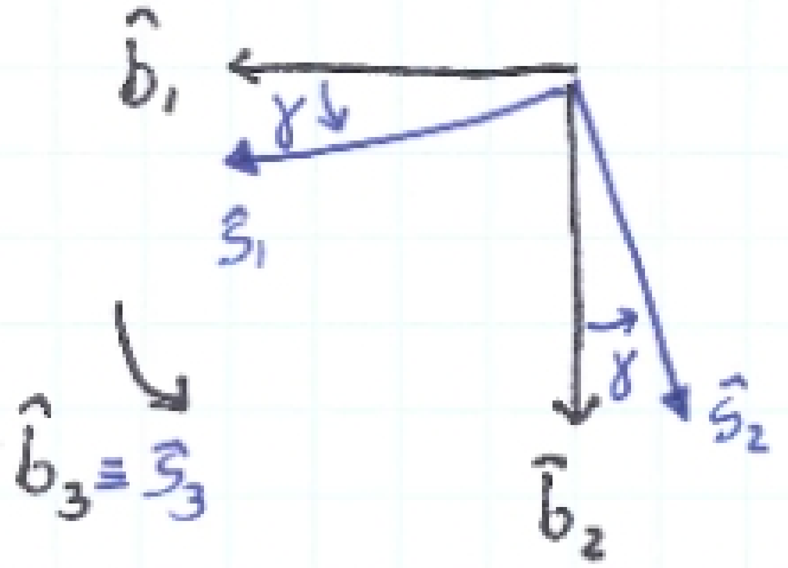
→ **ROT#2** $\quad {}^{\hat{a}}\bar{\omega}^{\hat{b}} = \dot{\sigma}\,\hat{a}_1 = \dot{\sigma}\,\hat{b}_1$ $\quad$ PITCH



	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$
$\hat{a}_1$	1	0	0
$\hat{a}_2$	0	c_σ	$-s_\sigma$
$\hat{a}_3$	0	s_σ	c_σ

$= [\ell]$
$\hat{a}\cdot\hat{b}$

$\boxed{\text{ROT \# 3}}$ $\quad {}^{b}\bar{\omega}^{s} = \dot{\gamma}\,\hat{b}_3 = \dot{\gamma}\,\hat{n}_3$ $\quad$ <u>ROLL</u>



$$\begin{array}{c|ccc} & \hat{s}_1 & \hat{s}_2 & \hat{s}_3 \\ \hline \hat{b}_1 & c\gamma & -s\gamma & 0 \\ \hat{b}_2 & s\gamma & c\gamma & 0 \\ \hat{b}_3 & 0 & 0 & 1 \end{array} = \underset{\overleftarrow{\hat{b}\cdot\hat{s}}}{[\ell]}$$

(b) From the Frames definition in part (a), we can immediately write

$$\underset{\overleftarrow{\hat{a}\cdot\hat{n}}}{[\ell]} = \begin{bmatrix} c\eta & 0 & -s\eta \\ 0 & 1 & 0 \\ s\eta & 0 & c\eta \end{bmatrix}$$

$$\boxed{\underset{\overleftarrow{\hat{n}\cdot\hat{a}}}{[\ell]} = \begin{bmatrix} c\eta & 0 & s\eta \\ 0 & 1 & 0 \\ -s\eta & 0 & c\eta \end{bmatrix}}$$

$$\boxed{\underset{\overleftarrow{\hat{b}\cdot\hat{a}}}{[\ell]} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\delta & +s\delta \\ 0 & -s\delta & c\delta \end{bmatrix}}$$