Name (PRINTED): ______________________________

Student ID #: ______________________________

Section # (or TA's: name and time) ______________________________

CMSC 250 Quiz #9 Wednesday, Mar. 31, 2004

Write all answers legibly in the space provided. The number of points possible for each question is indicated in square brackets – the total number of points on the quiz is 30, and you will have exactly 20 minutes to complete this quiz. You may not use calculators, textbooks or any other aids during this quiz.

1. [15 pnts.]Use regular induction to prove the following inequality.

$$\forall n \in Z \text{ where } n \geq 6, \;\; 4n < n^2 - 7$$

Base Case:$(n = 6)$ $4(6) = 24$ ================== $6^2 - 7 = 36 - 7 = 29$ ================== $24 < 29$
Inductive Hypothesis:$(n = x)$ $4x < x^2 - 7$
Inductive Step:$(n = x + 1)$ Show: $4(x + 1) < (x + 1)^2 - 7$
Proof: By the I.H., $4x < x^2 - 7$ Adding 4 to both sides, $4x + 4 < x^2 - 7 + 4$ Doing Algebra, $4(x + 1) < x^2 - 3$ Since we know that $0 < 2x - 3 \; \forall x \in Z^{\geq 2}$ We can add the 0 to the left and the $2x - 3$ to the right and get $4(x + 1) < x^2 - 3 + 2x - 3$ Doing Algebra, $4(x + 1) < x^2 + 2x + 1 - 7$ after further algrbra, $4(x + 1) < (x + 1)^2 - 7$ QED

2. [15 pnts.] Use strong induction to prove the following statement.

Assume:

$$a_1 = 3 \quad a_2 = 5 \quad a_3 = 1 \quad a_n = 2a_{n-1} + 3a_{n-2} + 4a_{n-3}$$

Prove that

$$\forall n \in Z \text{ where } n \geq 1, \;\; a_n \in Z^{odd}$$

Base Case:$(n = 1, 2, 3)$

$a_1 = 3, 3 \in Z^{odd}$

$a_2 = 5, 5 \in Z^{odd}$

$a_3 = 1, 1 \in Z^{odd}$

Inductive Hypothesis:$(n = i \forall i \in Z 4 \leq i \leq k)$

$a_i \in Z^{odd}$ where $a_i = 2a_{i-1} + 3a_{i-2} + 4ai - 3$

Inductive Step:$(n = k + 1)$

Show:

$a_{k+1} \in Z^{odd}$ where $a_{k+1} = 2a_k + 3a_{k-1} + 4a_{k-2}$

Proof:

By the I.H.:

Since $a_k \in Z^{odd}, \exists m \in Z \; a_k = 2m + 1$

Since $a_{k-1} \in Z^{odd}, \exists p \in Z \; a_{k-1} = 2p + 1$

Since $a_{k-2} \in Z^{odd}, \exists q \in Z \; a_{k-2} = 2q + 1$

By Substitution, $a_{k+1} = 2(2m + 1) + 3(2p + 1) + 4(2q + 1)$

$= 4m + 2 + 6p + 3 + 8q + 4$

$= 4m + 6p + 8q + 9$

$= 2(2m + 3p + 4q + 4) + 1$

Since $2m + 3p + 4q + 4 \in Z$,

$a_{k+1} \in Z^{odd}$ by the definition of odd. QED