

## Examples of Addition in Non-decimal Bases

**Example 1:** Compute  $(313)_{\text{four}} + (22)_{\text{four}}$  in base four.

I suggest you do this by way of a place value diagram. Set the problem up as follows

$$\begin{array}{r}
 4^2s \quad 4^1s \quad 4^0s \\
 3 \quad 1 \quad 3 \\
 + \quad \quad 2 \quad 2 \\
 \hline
 \end{array}$$

Notice this is the same as how I make a place value diagram for regular addition except instead of 100s, 10s and 1s, I use  $4^2$ s,  $4^1$ s, and  $4^0$ s as my place values.

Now add the columns up. If any number in your answer is bigger or equal to the base four put it in parentheses.

$$\begin{array}{r}
 4^2s \quad 4^1s \quad 4^0s \\
 3 \quad 1 \quad 3 \\
 + \quad \quad 2 \quad 2 \\
 \hline
 3 \quad 3 \quad (5)
 \end{array}$$

The numbers in parenthesis are *not* proper digits in base four so I must make some exchanges. Since the (5) in the  $4^0$  column is not a proper base four number, I must rewrite it. But 5 is 1 four and 1 one. So I add 1 in the  $4^1s$  column and replace the (5) in the ones column with a 1. This gives me

$$\begin{array}{r}
 4^2s \quad 4^1s \quad 4^0s \\
 3 \quad 1 \quad 3 \\
 + \quad \quad 2 \quad 2 \\
 \hline
 3 \quad 3 \quad (5) \\
 \hline
 3 \quad (4) \quad 1
 \end{array}$$

Now the (4) in the  $4^1s$  column is not a proper base four number so I must make an exchange. The digit 4 is one 4 and 0 ones so I add 1 to the  $4^2$  column and replace (4) with 0. This gives me

$$\begin{array}{r}
 4^2s \quad 4^1s \quad 4^0s \\
 3 \quad 1 \quad 3 \\
 + \quad \quad 2 \quad 2 \\
 \hline
 3 \quad 3 \quad (5) \\
 \hline
 3 \quad (4) \quad 1 \\
 \hline
 (4) \quad 0 \quad 1
 \end{array}$$

Now (4) is not a proper base four number so I must rewrite it for my final answer. The digit 4 is 1 four and 0 ones. So I insert a column for  $4^3s$  and add 1 to it then replace the (4) with a 0. This gives me

$$\begin{array}{r}
 4^2s \quad 4^1s \quad 4^0s \\
 3 \quad 1 \quad 3 \\
 + \quad \quad 2 \quad 2 \\
 \hline
 3 \quad (3) \quad (5) \\
 \hline
 3 \quad (4) \quad 1 \\
 \hline
 (4) \quad 0 \quad 1 \\
 \hline
 1 \quad 0 \quad 0 \quad 1
 \end{array}$$

So my answer is

$$(313)_{\text{four}} + (22)_{\text{four}} = (1001)_{\text{four}}.$$

**Example 2:** Compute  $(320)_{\text{four}} - (221)_{\text{four}}$  in base four.

I suggest you do this by way of a place value diagram. Set the problem up as follows

$$\begin{array}{r}
 4^2s \quad 4^1s \quad 4^0s \\
 3 \quad 2 \quad 0 \\
 - \quad 2 \quad 2 \quad 1 \\
 \hline
 \end{array}$$

Notice this is the same as how I make a place value diagram for regular subtraction except instead of 100s, 10s and 1s, I use  $4^2s$ ,  $4^1s$ , and  $4^0s$  as my place values.

Now we want to do the subtraction starting from the  $4^0s$  column, however we can't take 1 from 0 so

