Notes 10.

Thermohydrodynamic Bulk-Flow Model in Thin Film Lubrication

General flow characteristics in oil lubricated fluid film bearings

a) Incompressible liquids of large viscosity (mineral oils)
b) Dominance of shear driven (*Couette*) flow over pressure (*Poiseuille*) driven flow
c) Fluid inertia and flow turbulence are usually NOT important (low circumferential flow Reynolds numbers)
d) Heat transfer to bearing cartridge and to/from shaft are important along with mechanical deformations induced by temperature gradients
e) Fluid temperature gradient along axial plane is negligible
f) Thermal effects change the lubricant viscosity and operating clearance, thus affecting significantly to the bearing static load performance

Thermohydrodynamic analyses are important in heavily loaded hydrodynamic bearings such as pressure dam bearings, tilting pad bearings, etc. See Notes 7

General flow characteristics in process fluid film bearings

applicable to damper annular seals and hybrid (hydrostatic + hydrodynamic) bearings

a) Process liquids have low viscosity (water, R_{134}, LH_2, LOx)
b) Material compressibility important, low bulk modulues (LH_2)
c) Large pressure drops along axial direction with significant mass flow rates (annular damper seals & hydrostatic bearings – up to 6,000 psig in cryogenic turbopumps)
d) Large heat capacity for transport of energy along axial direction
e) Large rotor speeds (up to 100 krpm) will induce large shear flow energy dissipation
f) Typically use macro-textured surfaces (roughened stator) to avoid generation of cross-coupled stiffness and to promote dynamic stability
g) Inlet fluid flow circumferential swirl is important (for rotordynamic stability)

These operation characteristics determine the need to account for

a) Flow turbulence (induced by shaft rotation and pressure driven flow conditions)
b) Fluid inertia effects – temporal and advective types.
c) Fluid properties depend on pressure and temperature (needs equation of state)
d) Adequate physics based modeling of machined surface texture (roughness)
e) Two-phase flow conditions under certain operating regimes

Bulk-Flow Equations for Thin Fluid Films

The fluid flow within a thin film region, see Fig.1, is governed by the continuity (mass conservation), momentum and energy transport equations. In the flow region $\{x \in (0, \pi D), y \in (0, H(x,z,t)), z \in (0, L)\}$, the smallness of the film thickness allows a simplification of the general transport equations.

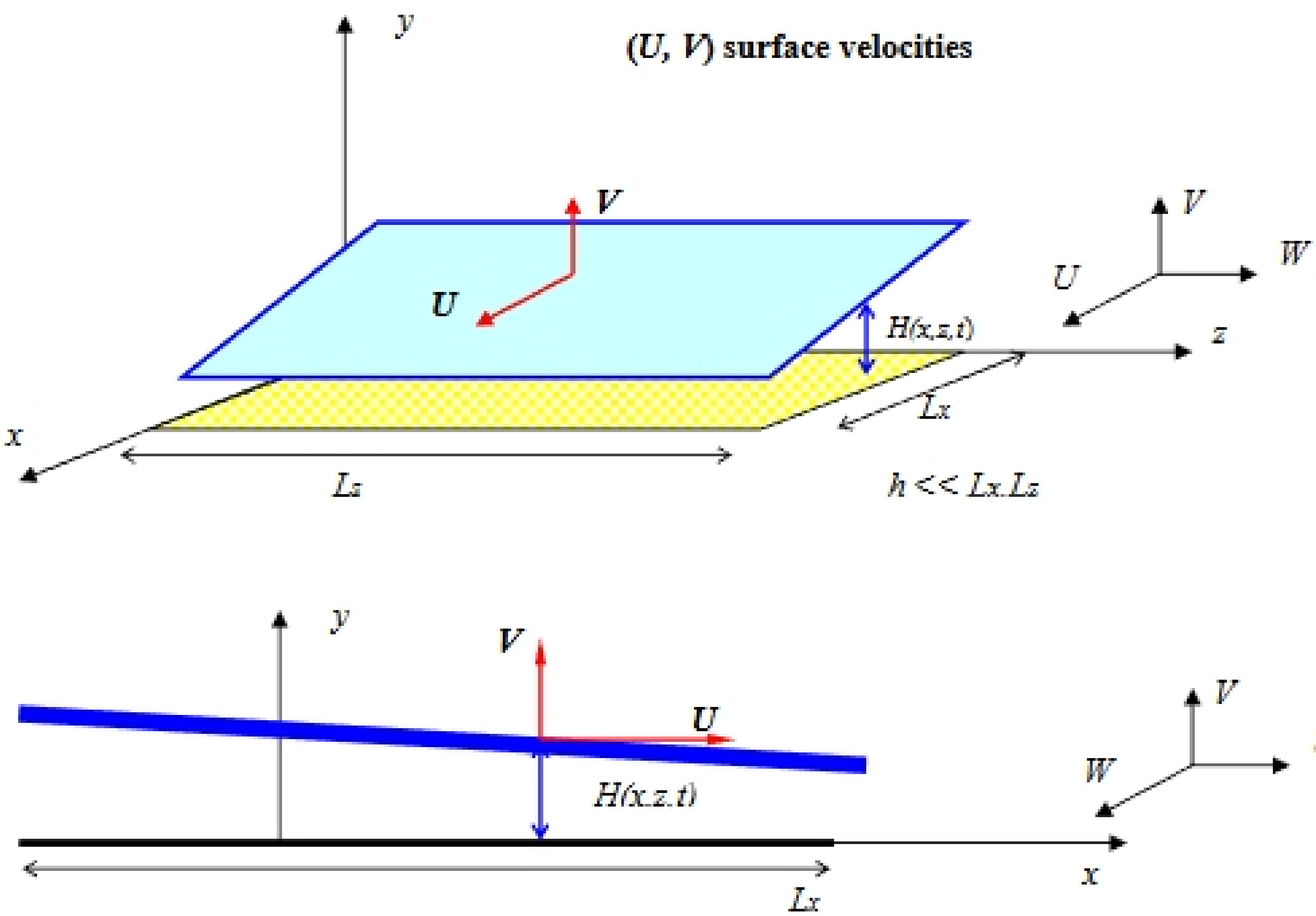

Figure 1. Geometry of flow region in a fluid film bearing *(H<<L_x,L_z)*

The coordinates in the plane of the bearing are circumferential ($x=R\theta$), axial (z), and across the film (y). Let $\{\tilde{U}, \tilde{V}, \tilde{W}\}, \tilde{P}, \tilde{T}$ be the fluid velocity field components along the (x, y, z) directions, the fluid pressure and its temperature, respectively.

The thin film fluid flow equations are (see Notes 8):

Mass conservation
$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho \tilde{U})}{\partial x} + \frac{\partial (\rho \tilde{V})}{\partial y} + \frac{\partial (\rho \tilde{W})}{\partial z} = 0 \qquad (1)$$

Circumferential-momentum transport

$$\rho \frac{D\tilde{U}}{Dt} = -\frac{\partial \tilde{P}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \tag{2}$$

Axial-momentum transport

$$\rho \frac{D\tilde{W}}{Dt} = -\frac{\partial \tilde{P}}{\partial z} + \frac{\partial \tau_{zy}}{\partial y} \tag{3}$$

Cross film momentum

$$\frac{\partial \tilde{P}}{\partial y} = 0 \tag{4}$$

Energy-Transport *(*Bird et Al., 1960*)*

$$\rho C_p \frac{D\tilde{T}}{Dt} + \frac{\rho}{2}\frac{D}{Dt}\left(\tilde{U}^2 + \tilde{W}^2\right) = \frac{\partial}{\partial y}\left(K\frac{\partial \tilde{T}}{\partial y}\right) + \tilde{T}\beta_t \frac{D\tilde{P}}{Dt} - \left(\tilde{U}\frac{\partial \tilde{P}}{\partial x} + \tilde{W}\frac{\partial \tilde{P}}{\partial z}\right) + \frac{\partial}{\partial y}\left(\tilde{U}\tau_{xy} + \tilde{W}\tau_{zy}\right) \tag{5}$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \tilde{U}\frac{\partial}{\partial x} + \tilde{V}\frac{\partial}{\partial y} + \tilde{W}\frac{\partial}{\partial z} \tag{6}$$

is the material derivative. $\{\rho, \mu, C_p, k, \beta_t\}$ represent the fluid properties of density, viscosity, specific heat, thermal conductivity, and volumetric expansion coefficient, respectively.

In a turbulent flow, the effect of the turbulent mixing far outweighs the fluid molecular diffusivity. In consequence, the temperature raise by viscous dissipation tends to be distributed uniformly across the film thickness. Thus, temperature gradients across the film (*y*-dir) are confined to (very thin) boundary layers attached to the bearing and journal surfaces. The fluid velocity field presents the same characteristics in regions without reversed flow or recirculation.

Bulk-flow primitive variables (velocities and temperature) represent average quantities across the film thickness, i.e.,

$$U = \frac{1}{H}\int_0^H \tilde{U}dy;\; W = \frac{1}{H}\int_0^H \tilde{W}dy;\; T = \frac{1}{H}\int_0^H \tilde{T}dy \tag{7}$$

Integration of Eqs. (1-5) across the film thickness renders the **bulk-flow equations (fully developed condition)**:

Continuity:

$$\frac{\partial(\rho H)}{\partial t} + \frac{\partial(\rho H U)}{\partial x} + \frac{\partial(\rho H W)}{\partial z} = 0 \tag{8}$$