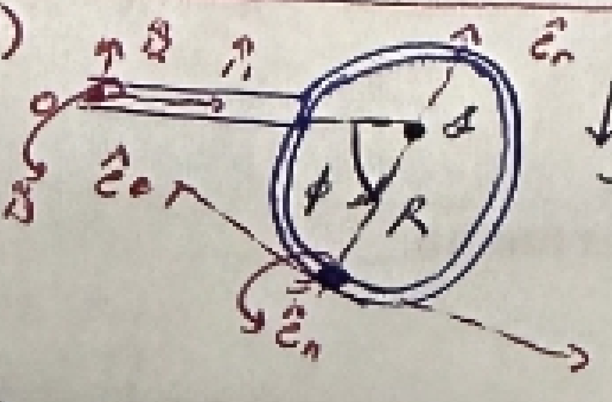
AAE 340 E2 1R1

AAE 340 FA18 E2

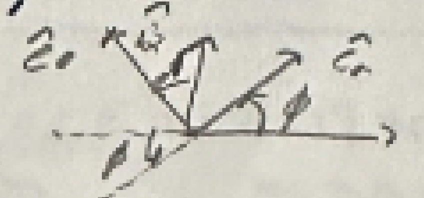
(2)



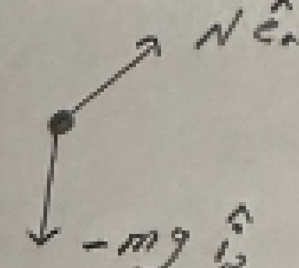
Let frame i be inertially fixed at point O and let e frame be fixed at point Δ,

${}^{i}\vec{\omega}^{e} = \dot{\phi}\,\hat{e}_n = \dot{\phi}\,\hat{i}_3$, $\bar{R}^{\Delta P} = -R\,\hat{e}_r$

$\hat{e}_r,\ \hat{e}_\theta,\ \hat{e}_n$



(a) FBD;



(b) compute the work;

$W = T_0(t_2) - T_0(t_1) = \frac{1}{2}m|{}^{i}\vec{v}^{OP}|^2\Big|_{t_1}^{t_2}$

$\bar{R}^{OP} = -R\,\hat{e}_r$; ${}^{i}\vec{v}^{OP} = ({}^{i}\vec{\omega}^{e} \times \bar{R}^{OP})$

${}^{i}\vec{v}^{OP} = (\dot{\phi}\,\hat{e}_n \times -R\,\hat{e}_r) = -R\dot{\phi}\,\hat{e}_\theta$

(b) $W = \int_{t_1}^{t_2} F_p \cdot {}^{i}\vec{v}^{OP}\,dt = \int_{t_1}^{t_2} (N\hat{e}_r \cdot -R\dot{\phi}\,\hat{e}_\theta)\,dt = \boxed{0}$ b/c the normal force is a constraint force $\perp$ to the path.

gravity is a conservative force b/c there is a scalar potential function: mgh. Work by gravity: $W = \int_{t_1}^{t_2} (-mg(c_\phi\,\hat{e}_\theta + s_\phi\,\hat{e}_r)) \cdot (-R\dot{\phi}\,\hat{e}_\theta)\,dt$

$W = \int_{t_1}^{t_2} mg\,c_\phi R\dot{\phi}\,dt = \int_{\phi_1}^{\phi_2} mgR\,c_\phi\,d\phi = \boxed{mgR\,s_\phi\Big|_{\phi_1}^{\phi_2} = W}$

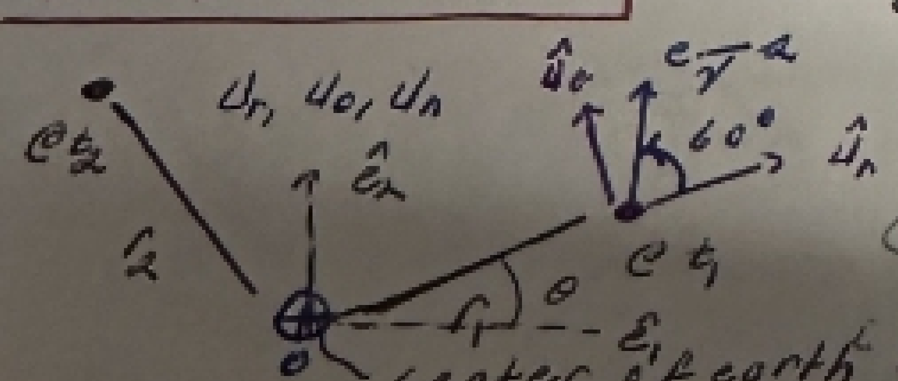
OR $h = R_{OP} \cdot \hat{i}_2 = -R\,\hat{e}_r \cdot (c_\phi\,\hat{e}_\theta + s_\phi\,\hat{e}_r)$

$\phi_1 = \phi(t_1)$. $\phi_2 = \phi(t_2)$

(c) $D = E(t_2) - E(t)$, what is D expected to be?

D should be zero or extremely close b/c there are no non conservative forces and total energy must be therefore conserved.

E2 PRACTICE 2



(a) FBD;

$F_g = \dfrac{-G\,m_s\,m_e}{r_1^2}\,\hat{u}_r$

(b) $\bar{R}^{OP} = r_1\,\hat{u}_r$; ${}^{i}\vec{v}^{OP} = \dot{r}\,\hat{u}_r + r\dot{\theta}\,\hat{u}_\theta$

${}^{i}\vec{a}^{OP} = \ddot{r}\,\hat{u}_r + (\dot{r}\dot{\theta} + r\ddot{\theta})\,\hat{u}_\theta + [(\dot{\theta}\,\hat{u}_n) \times {}^{i}\vec{v}^{OP}]$

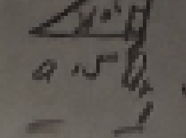
${}^{i}\vec{a}^{OP} = \ddot{r}\,\hat{u}_r + (\dot{r}\dot{\theta} + r\ddot{\theta})\,\hat{u}_\theta + \dot{\theta}\dot{r}\,\hat{u}_\theta - r\dot{\theta}^2\,\hat{u}_r$

${}^{i}\vec{a}^{OP} = (\ddot{r} - r\dot{\theta}^2)\,\hat{u}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\,\hat{u}_\theta$. $F_{ext} = m_s\,{}^{i}\vec{a}^{OP}$

$-\dfrac{G m_e}{r_1^2}\,\hat{u}_r = (\ddot{r} - r\dot{\theta}^2)\,\hat{u}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\,\hat{u}_\theta$

(b) $\ddot{r} - r\dot{\theta}^2 = -\dfrac{\mu}{r_1^2}$, $r\ddot{\theta} = -2\dot{r}\dot{\theta}$ $\boxed{\ddot{r} - r\dot{\theta}^2 = -\dfrac{\mu}{r_1^2}}$ $\boxed{\ddot{\theta} + \dfrac{2\dot{r}\dot{\theta}}{r} = 0}$

(c) $r_1 = 6771\ km$ $r_2 = 6871\ km$;



$\frac{5}{2}\sqrt{3}\,\hat{u}_\theta$ $25 = v_r^2 + v_\theta^2$

h_0 = constant

$E(t_1) = E(t_2)$, $\frac{1}{2}\cancel{m}|{}^{e}v_1^{\,s}|^2 - \dfrac{\mu \cancel{m_s}}{r_1} = \frac{1}{2}\cancel{m}|{}^{e}v_2^{\,s}|^2 - \dfrac{\mu \cancel{m_s}}{r_2}$

$r_1 v_{\theta_1} = r_2 v_{\theta_2}$

$\frac{1}{2}(25) - \dfrac{\mu}{6771} = \frac{1}{2}|v_2^2| - \dfrac{\mu}{6871}$ $v_2^2 = 23.2864 = v_r^2 + v_\theta^2$, $v_r = 2.2523$ $v_\theta = \frac{r_1}{r_2} v_\theta = 4.268$

$\boxed{v_2 = 2.2523\,\hat{u}_r + 4.268\,\hat{u}_\theta}$