AMS 572 Lecture Notes #2

Sept. 9, 2011.

Chapter 7: Inference on one population mean □

1 Motivation & simple random sample

Eg) We wish to estimate the average height of adult US males

➔ Take a random sample.

- "Simple" random sample: every subject in the population has the same chance to be selected.

2 Introduction to statistical inference on one population mean

For a "**random** sample" of size n: $X_1, X_2, \ldots, X_n$

<i> Point estimatior $\bar{X}$ **➔ sample mean (** $= \frac{X_1 + X_2 + \ldots + X_n}{n} = \frac{\sum_{i=1}^{n} X_i}{n}$ **)**

Other estimators: median, mode, trimmed mean, …

<ii> **Confidence Interval (C.I.)**

Eg) 95% C.I. for μ

99.9999% C.I. ('6-9' in the manufacture industry)

<iii> **Hypothesis Test**

Eg) H_0 : μ [illegible]

H_1 : μ [illegible]

Point Estimator, C.I., Test ⇒ Statistical Inference

- Draw some conclusion on the population (parameters of interest) based on a random sample.

3 Normal Distribution

<i> Probability Density Function (p.d.f.)

$X \sim N(\mu, \sigma^2)$

(X follows normal distribution of mean μ and variance σ^2)

$$f(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad , \quad -\infty < x < \infty, x \in R$$

$P(a \le X \le b)=\int_a^b f(x)dx$ = area under the pdf curve bounded by a and b

<ii> Cumulative Density Function (c.d.f.)

$$F(x)=P(X \le x)=\int_{-\infty}^{x} f(t)dt$$

$$f(x)=[F(x)]'=\frac{d}{dx}F(x)$$

<iii> Z-score

Standard Normal Distribution

$Z \sim N(0,1)$

$X \sim N(\mu, \sigma^2)$

$$Z=\frac{X-\mu}{\sigma} \sim N(0,1)$$

$$P(a \le X \le b)=P(\frac{a-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{b-\mu}{\sigma})$$

$$=P(\frac{a-\mu}{\sigma} \le Z \le \frac{b-\mu}{\sigma})$$

Proof) 1> Use c.d.f.

$$F_Z(z)=P(Z \le z)=P(\frac{X-\mu}{\sigma} \le z)$$

¿ $P(X \le \sigma \cdot z+\mu)=F_X(\sigma \cdot z+\mu)$

$$f_Z(z)=\frac{d}{dz}F_Z(z)=\frac{d}{dz}F_X(\sigma \cdot z+\mu)=f_X(\sigma \cdot z+\mu)\cdot\sigma$$

$$=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(\sigma\cdot z+\mu-\mu)^2}{2\sigma^2}}\cdot\sigma=\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}$$ → the p.d.f. for N(0,1)

2> Use p.d.f.

$$f_X(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$x=\sigma\cdot z+\mu$$

$$|J|=\frac{dx}{dz}=\frac{d}{dz}(\sigma\cdot z+\mu)=\sigma$$

$$f_z(z)=|J|\cdot f_x(x)=\sigma\cdot\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}=\frac{1}{\sqrt{2\pi}}e^{-\frac{(\sigma\cdot z+\mu-\mu)^2}{2\sigma^2}}=\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}$$

$$\therefore Z=\frac{X-\mu}{\sigma}\sim N(0,1)$$

3> Use m.g.f.

$$M_X(t)=E(e^{tX})$$

$$=\int_{-\infty}^{\infty}e^{tx}f(x)dx$$, if x is continuous

$$=\sum_{x's}e^{tx}f(x)$$, if x is discrete

Theorem If two 'RV's have the same MGF, they are of the same distribution.

Normal Distributions $X\sim N(\mu,\sigma^2)$

$$M_X(t)=e^{\mu\cdot t+\frac{1}{2}\sigma^2t^2}$$

Eg) If the mgf of X is $e^{-7t+10t^2}$, then $X\sim N(-7,20)$

$$Z=\frac{X-\mu}{\sigma},X\sim N(\mu,\sigma^2)\Rightarrow M_Z(t)$$?

Linear transformation : $Y=a\cdot X+b$, a&b are constants

$$M_Y(t)=E(e^{tY})=E[e^{t(aX+b)}]=E(e^{atX+bt})=E(e^{atX}\cdot e^{bt})$$