AMS 572 Lecture Notes #5

September 23, 2011

<*Today's Topic*>

Power Calculation (Inference on one population mean)

		Truth	
		H_0	H_a
Decision	H_0		***Type II error***
	H_a	***Type I error***	

β = P(Type II error) = P(Fail to reject H_0 | H_a)

⑦ Power Calculation (Inference on one population mean)

1. $H_0: \mu = \mu_0$
$H_a: \mu > \mu_0 \ (\Rightarrow \mu = \mu_a > \mu_0)$

1st scenario, Normal population, σ^2 is known.

Test statistic : $Z_0 = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \overset{H_0}{\sim} N(0,1)$

At the significance level α , we reject H_0 if $Z_0 \geq Z_\alpha$

Power of the test = P(reject H_0 | H_a)

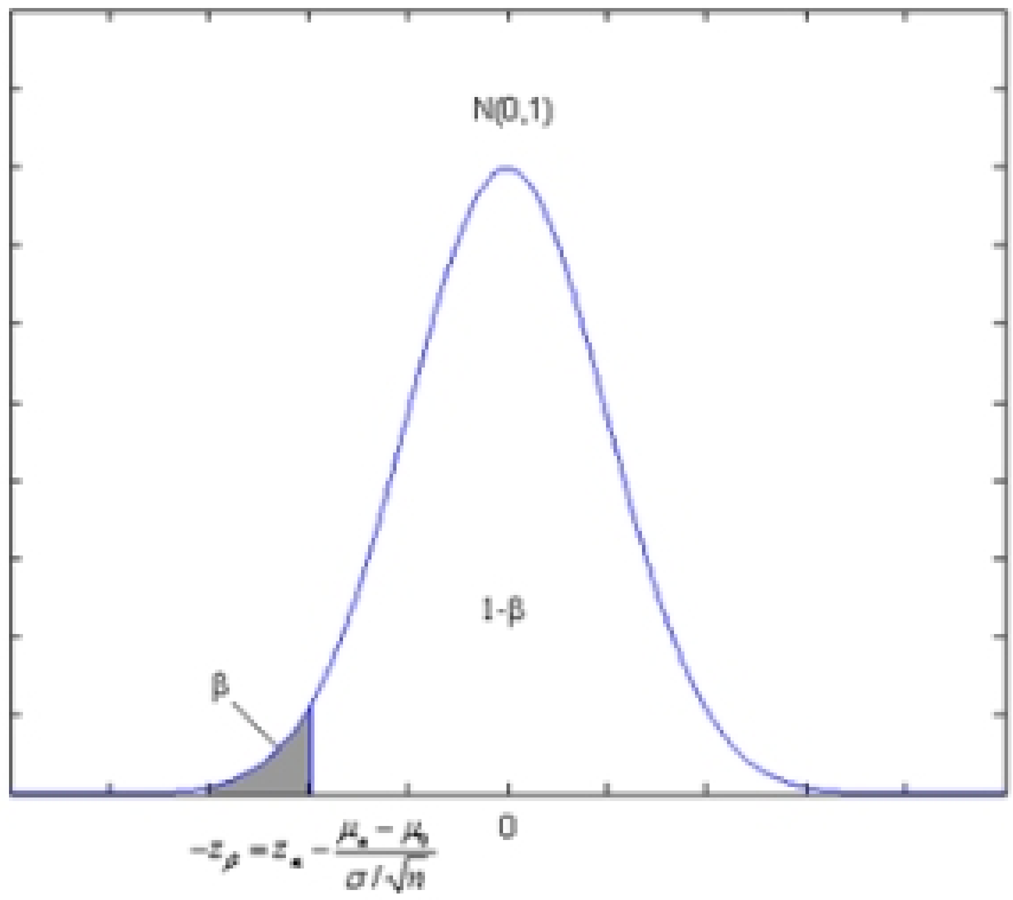
= 1- β

Power = 1- β = P(reject H_0 | H_a) = P($Z_0 \geq Z_\alpha | \mu = \mu_a)$

= $P(\dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \geq Z_\alpha | \mu = \mu_a)$, If $\mu = \mu_a$, $Z_0 = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(\dfrac{\mu_a - \mu_0}{\sigma/\sqrt{n}}, 1)$

$$= P\left(\frac{\overline{X}-\mu_a}{\sigma/\sqrt{n}}+\frac{\mu_a-\mu_0}{\sigma/\sqrt{n}}\geq Z_\alpha \mid \mu=\mu_a\right)$$

$$= P\left(Z\geq Z_\alpha-\frac{\mu_a-\mu_0}{\sigma/\sqrt{n}} \mid \mu=\mu_a\right), \quad Z\sim N(0,1)$$



2nd scenario, Normal population, σ^2 is unknown.

Test statistic : $T_0=\frac{\overline{X}-\mu_0}{S/\sqrt{n}} \overset{H_0}{\sim} t_{n-1}$

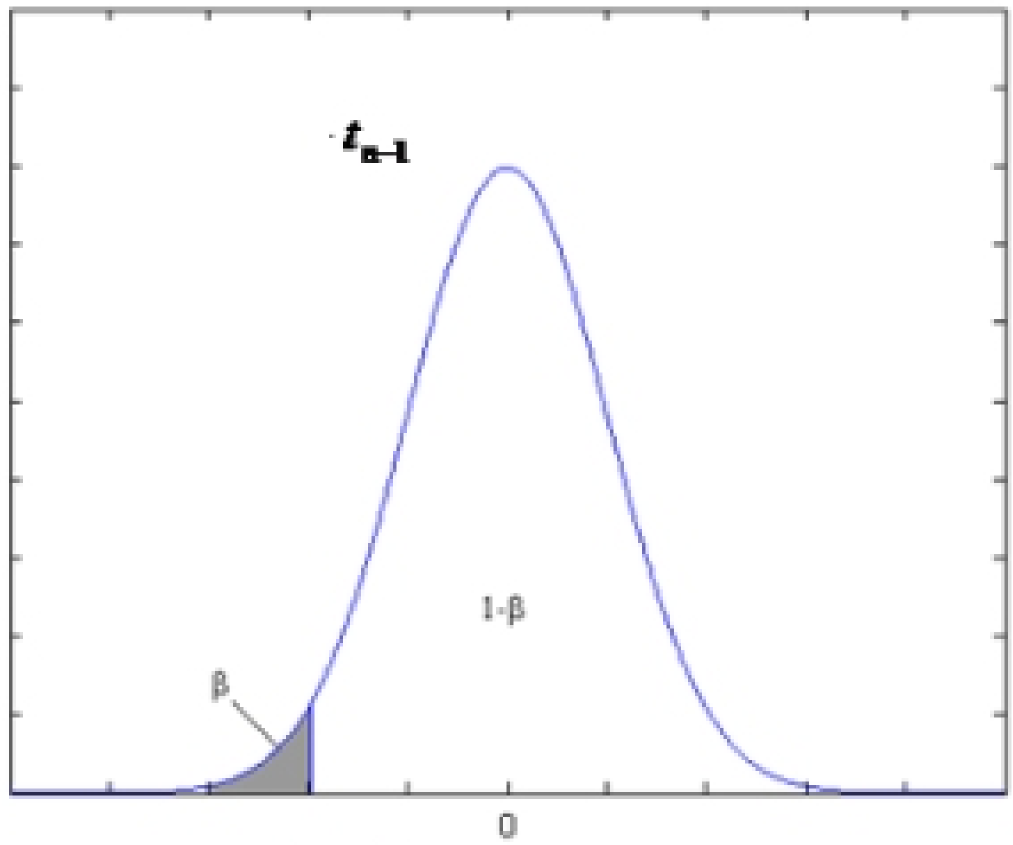
At the significance level α , we reject H_0 if $T_0\geq t_{n-1,\alpha}$

Power of the test = P(reject H_0 | H_a) = 1- β

Power = 1- β = P(reject H_0 | H_a) = P($T_0\geq t_{n-1,\alpha} \mid \mu=\mu_a)$

$$= P\left(\frac{\overline{X}-\mu_0}{S/\sqrt{n}}\geq t_{n-1,\alpha} \mid \mu=\mu_a\right) = P\left(\frac{\overline{X}-\mu_a}{S/\sqrt{n}}+\frac{\mu_a-\mu_0}{S/\sqrt{n}}\geq t_{n-1,\alpha} \mid \mu=\mu_a\right)$$

$$= P\left(T\geq t_{n-1,\alpha}-\frac{\mu_a-\mu_0}{S/\sqrt{n}} \mid \mu=\mu_a\right)$$, Here $T\sim t_{n-1}$



$$t_{n-1,\alpha} - \frac{\mu_a - \mu_0}{S/\sqrt{n}}$$

Recall that the ***Shapiro-Wilk test****: can be used to determine whether the population is normal.*

<u>3rd scenario, Any population (*usually we use this test when the population is found not normal), large sample (n ¿ 30)</u>

<u>Test statistic</u> : $Z_0 = \frac{\overline{X} - \mu_0}{S/\sqrt{n}} \overset{H_0}{\dot{\sim}} N(0,1)$

At the significance level α , we reject H_0 if $Z_0 \geq Z_\alpha$

<u>Power</u> of the test = P(reject H_0 | H_a) = 1- β

Power = 1- β = P(reject H_0 | H_a) = P($Z_0 \geq Z_\alpha | \mu = \mu_a)$

$= P\left(\frac{\overline{X} - \mu_0}{S/\sqrt{n}} \geq Z_\alpha | \mu = \mu_a\right) = P\left(\frac{\overline{X} - \mu_a}{S/\sqrt{n}} + \frac{\mu_a - \mu_0}{S/\sqrt{n}} \geq Z_\alpha | \mu = \mu_a\right)$

$= P\left(Z \geq Z_\alpha - \frac{\mu_a - \mu_0}{S/\sqrt{n}} | \mu = \mu_a\right)$, $Z \sim N(0,1)$