LESSON #22: FORMAL PROOF RULES FOR CONDITIONALS (8.2)

POWERPOINT SLIDE #1

There are some very important **equivalences** involving conditionals that you need to know on **p. 200** of the text. Keep in mind that these equivalences are *not* proof rules but instead are the sort of relations that could *be proven* by means of a formal proof.

$P \rightarrow Q \quad \Leftrightarrow \quad \neg Q \rightarrow \neg P$

The **Law of *Contraposition***; the second conditional here is called the *contrapositive* of the first. Note that this equivalence mirrors the inference pattern *modus tollens* that we learned in the last lesson.

Btw, *modus tollens* is short for the Latin phrase *modus tollendo tollens*, which means "the way that denies by denying". By 'denial' it refers to the negated forms found in the contrapositive.

Similarly, the other similar valid inference pattern *modus ponens* that we talked about last time is short for *modus ponendo ponens*, which means "the way that affirms by affirming". It 'affirms' in the sense that the inference is derived without any negations. In that sense, then, *modus tollens* is the 'negative' version of *modus ponens*.

$P \rightarrow Q \quad \Leftrightarrow \quad \neg P \vee Q$

The first disjunct here covers the cases where the antecedent of the conditional is false ($\neg P$) and thus doesn't trigger the conditional relation; the second disjunct covers the other case where the conditional statement is true, namely, when the condition is triggered by a true antecedent and thus yields a true consequent (Q). (Refer to the truth table for the material conditional in the previous lesson.)

POWERPOINT SLIDE #2

$\neg(P \rightarrow Q)$	⇔	$P \wedge \neg Q$	⇦ Note that this equivalence mirrors the second row of the truth table for the material conditional (where the conditional is *false*).
$P \leftrightarrow Q$	⇔	$(P \rightarrow Q) \wedge (Q \rightarrow P)$	⇦ This equivalence reflects one way of understanding the biconditional: as a two-way conditional relationship.
$P \leftrightarrow Q$	⇔	$(P \wedge Q) \vee (\neg P \wedge \neg Q)$	⇦ This equivalence reflects another way of understanding the biconditional: as the covariance of truth values.

There are two proof rules for the material conditional and two for the biconditional:

POWERPOINT SLIDE #3

<u>Conditional Elimination</u>: **→ Elim**
aka ***modus ponens***

If you know **P → Q** and also **P** individually, then you can assert **Q** individually as well (because P→ Q instructs you that Q follows from P).

1	**P → Q**	
2	**P**	
3	**Q**	**→ Elim: 1, 2**

POWERPOINT SLIDE #4

Conditional Introduction: **→ Intro**
aka **'Conditional Proof'**

If you can prove within a subproof that premising **P** leads to the conclusion **Q**, then you can discharge the subproof and assert **P → Q** one level back, citing the entire subproof for justification.

1	│ │ **P**	
2	│ │ **Q**	
3	│ **P → Q**	**→ Intro: 1-2**

POWERPOINT SLIDE #5

Note that the ***only way to prove a conditional statement*** such as **P → Q** (whether it's the ultimate goal of your proof or an intermediate goal that you need to reach before you can reach the ultimate goal) is to create a subproof of the sort we just saw which opens by premising the antecedent of the conditional statement you want to prove and concludes with the consequent of that conditional. So, anytime the goal of your proof is a conditional statement, you can go ahead and sketch in this overall structure, then worry about how to get from **P** to **Q**.

POWERPOINT SLIDE #6

Biconditional Elimination: **↔ Elim**

Basically, if you have a biconditional involving **P** and **Q** (i.e., **P ↔ Q** or **Q ↔ P**), and you also have one of the components **P** or **Q** individually, then you can conclude the *other* component (**P** or **Q**) individually as well. For example:

1	│ **P ↔ Q**	
2	│ **Q**	
3	│ **P**	**↔ Elim: 1, 2**