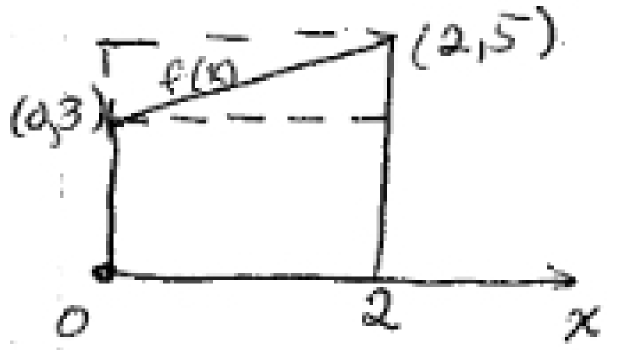
5.1

The Area under a non-negative function over $[a, b]$.

Introduction: We will approximate the area of the trapezoid shown using rectangles.



The lower rectangle has area base × ht = 6

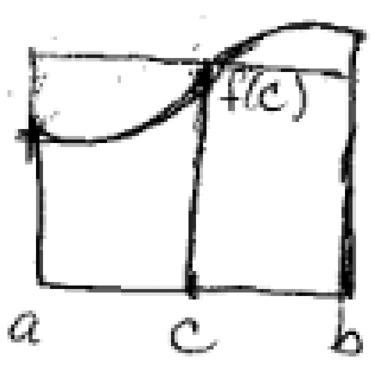
The upper rectangle has area 10.

The trapezoid area then is between 6 and 10.

The lower rectangle has height 3 which is the value on the function $f(x)$ at the left end, $f(0) = 3$. We call this a <u>left-hand rectangle</u>.
The upper rectangle has height $5 = f(2)$ and is a <u>right-hand rectangle</u>.

The true area of the trapezoid is actually $8 = \frac{10+6}{2}$ the average of the

left hand and right hand rectangle areas.



In general we can choose any c in $[a, b]$, form the rectangle with base $= b - a$ and height $f(c)$ and estimate the area as

$$\text{Area} \approx f(c)(b-a)$$

Calculus Concepts & Contexts,
James Stewart

From Figures 8 and 9 it appears that, as n increases, both L_n and R_n become better and better approximations to the area of S. Therefore we *define* the area A to be the limit of the sums of the areas of the approximating rectangles, that is,

TEC In Visual 5.1 you can create pictures like those in Figures 8 and 9 for other values of n.

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} L_n = \frac{1}{3}$$

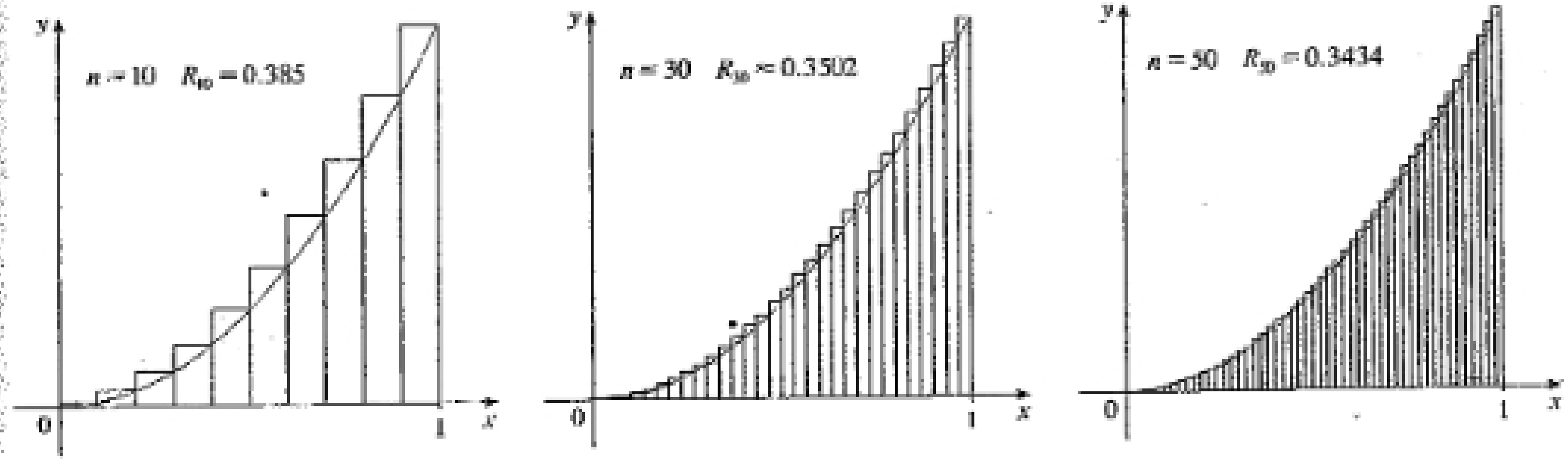


FIGURE 8

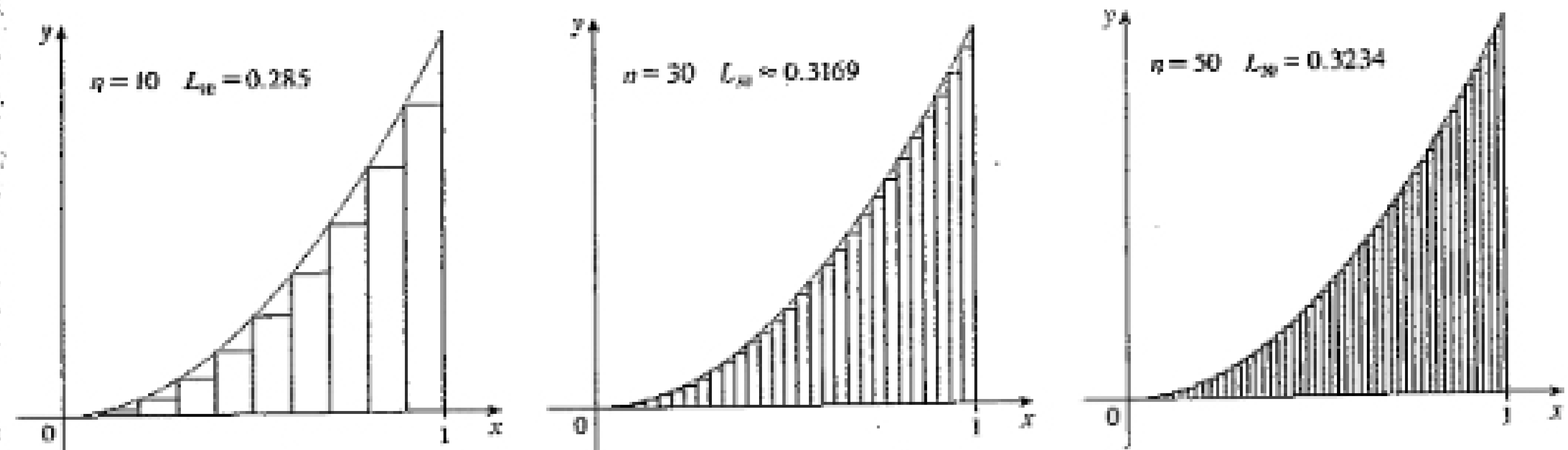


FIGURE 9 The area is the number that is smaller than all upper sums and larger than all lower sums

Let's apply the idea of Examples 1 and 2 to the more general region S of Figure 1. We start by subdividing S into n strips $S_1, S_2, \ldots, S_n$ of equal width as in Figure 10.

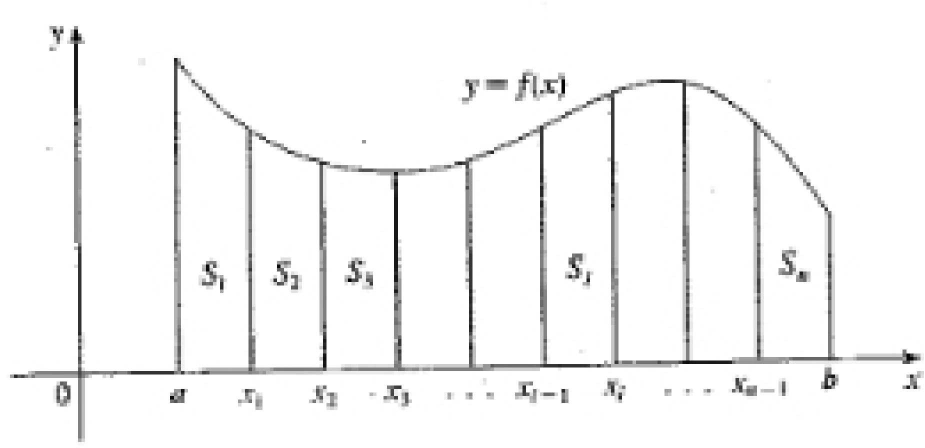


FIGURE 10

We get a still better estimate by partitioning $[a, b]$ into many small intervals and forming many thin rectangles and adding their areas.

$a = x_0 < x_1 < x_2 < x_3 < b = x_4$ Notation for a partition with 4 subintervals.

Example: Partition the interval $[1, 5]$ into 8 equal subintervals. Then each small interval has length $\frac{5-1}{8} = \frac{1}{2}$. Partition pts.

$1 \quad 2 \quad 3 \quad 4 \quad 5$ start at 1 and occur every $\frac{1}{2}$ unit up to 5.

$1 = x_0 < 1.5 < 2 < 2.5 < 3 < 3.5 < 4 < 4.5 < 5 = x_8$

Suppose $f(x) = x^3$. In each small interval we must choose an x-value to plug into $f(x) = x^3$ to form the height of that rectangle. If we choose the left endpoint every time we form the Left-Hand-Sum = LHS.
(Sum of rectangle areas formed at left endpts)
Similarly, choosing the right endpoint every time we form the Right-Hand-Sum = RHS.

To finish this exampe:

Add up $\frac{1}{2} \times f(c$
sum of base×ht

$$LHS = \frac{1}{2}\left[f(1) + f(1.5) + \ldots + f(4.5)\right]$$

$$= \frac{1}{2}\left[1^3 + 1.5^3 + \ldots + (4.5)^3\right] = 126.5$$

$$RHS = \frac{1}{2}\left[1.5^3 + 2^3 + \ldots + 5^3\right] = 188.5$$