Recitation 11 Notes

Context Free Grammars

<u>Definition:</u> A grammar G = (V, T, P,S) is a *context free grammar (cfg)* if all productions in P have the form

$$A \rightarrow x$$

where

- $A \in V$, and
- $x \in (V \cup T)^*$.

<u>Examples</u>

Problem 1.
Given the language $L = \{a^n b^n : n \geq 0\}$, what is the CFG that generates the language?

$G = (\{S\}, \{a, b\}, \{S \rightarrow aSb, S \rightarrow \varepsilon\}, S\}$

Problem 2.
Given the language $L = \{a^n b^k : k > n \geq 0\}$, what is the CFG that generates the language?

$G = (\{S, B\}, \{a, b\}, \{S \rightarrow aSb, S \rightarrow B, B \rightarrow bB, B \rightarrow b\}, S)$.

Problem 3.
The language $L = \{w: w \in \{a, b\}^*, n_a(w) = n_b(w)\}$, what is the CFG that defines the language?

$G = (\{S\}, \{a, b\}, \{S \rightarrow aSb, S \rightarrow bSa, S \rightarrow SS, S \rightarrow \varepsilon\}, S)$.

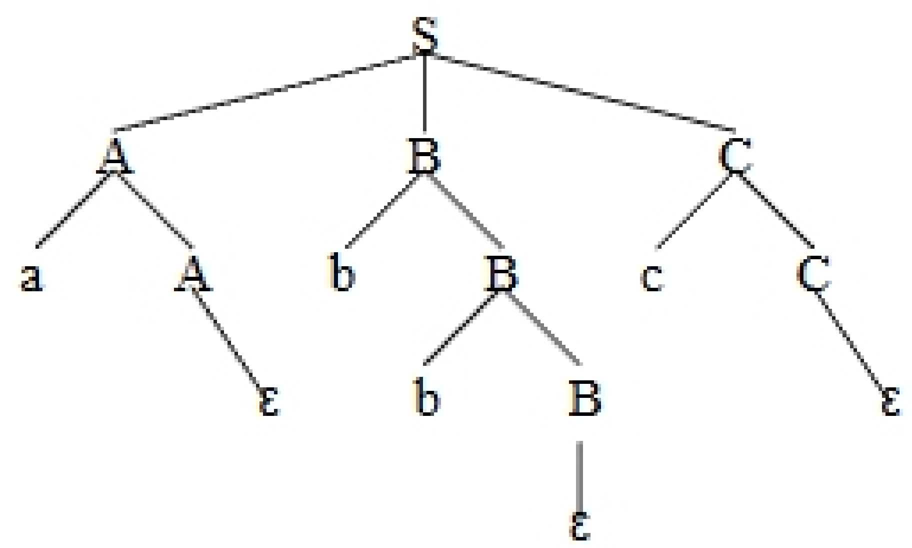
What is the PDA that recognizes this language?

1. Get input symbol
2. If input symbol is an 'a'

- Go to Step 1.

Derivation Trees

Consider Derivation 1, the tree can be constructed as follows:



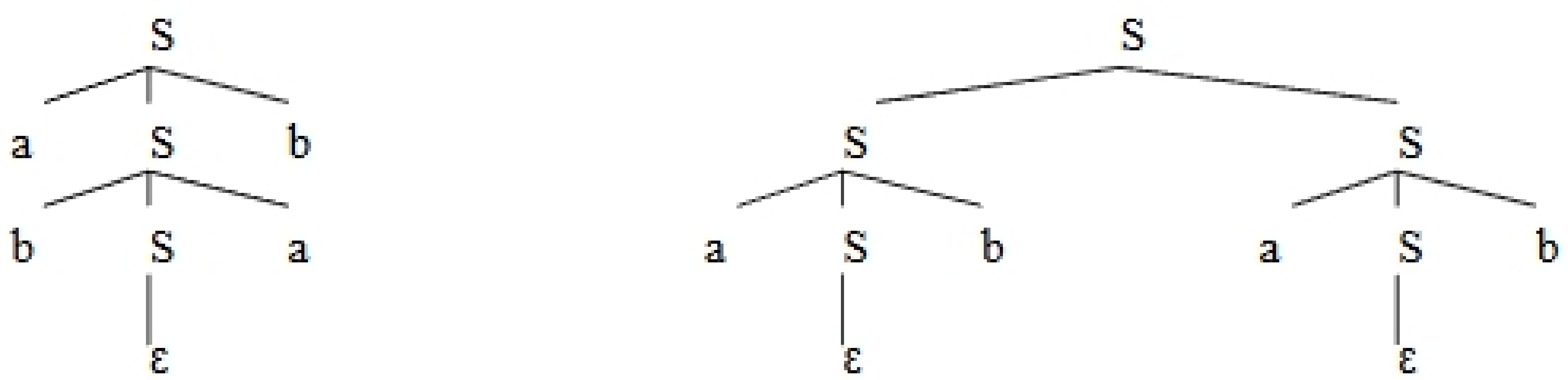
The *yield* of the tree is the final string obtained by reading the leaves of the tree from left to right, ignoring the ε's (unless *all* the leaves are ε, in which case the yield is ε). The yield of the above tree is the string abbc, as expected.

Ambiguity

Consider the grammar,

G = ({S}, {a, b}, {S→aSb, S→bSa, S→SS, S→ε},S).

'abab' can be derived from the grammar with two parse trees.



Definition: A grammar G is *ambiguous* if there exists some string w ∈ L(G) for which

- there are two or more distinct derivation trees, or
- there are two or more distinct leftmost derivations, or
- there are two or more distinct rightmost derivations.

Definition: An *inherently ambiguous language* is a language for which no unambiguous grammar exists.