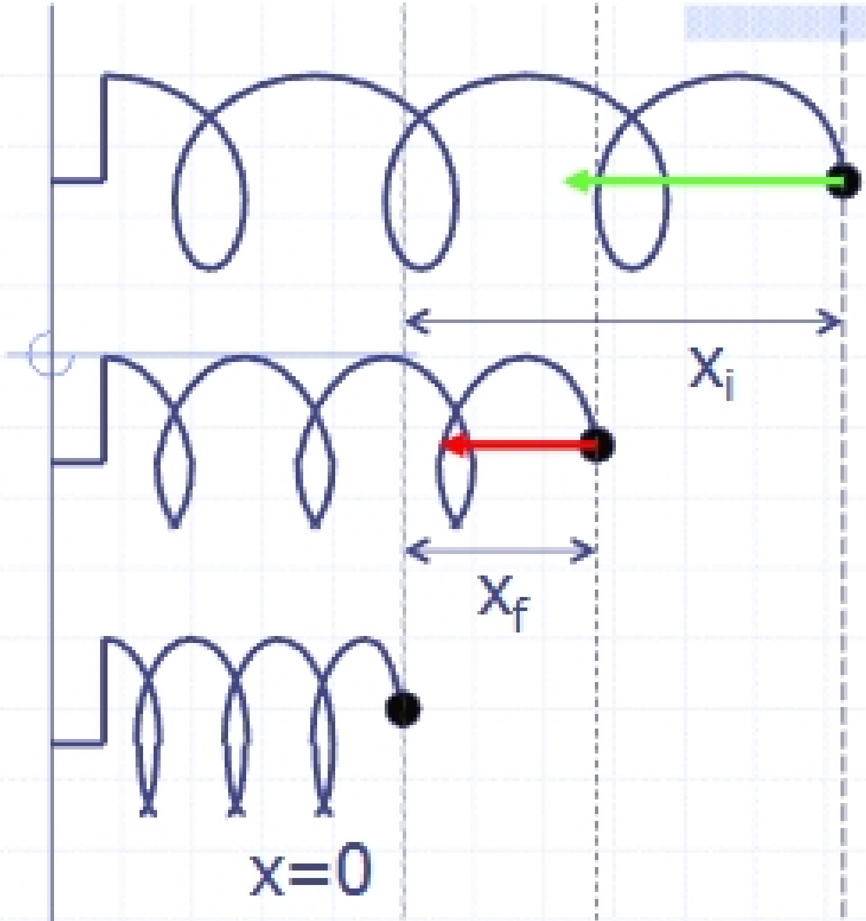
Work done by a spring

- We know that work equals force times displacement
- But how to we calculate the work due to a non-constant force?
- Reconsider the restoring force of a spring

$$F_s = -kx$$

Hooke's Law for the restoring force of an ideal spring.

- It depends on the distance the spring is stretched or compressed.



$$d = x_f - x_i$$

But the force is not constant

$$F_{s,i} = -kx_i,$$

$$F_{s,f} = -kx_f$$

Take the average force

$$F_{s,avg} = \frac{F_{s,i} + F_{s,f}}{2}$$

$$F_{s,avg} = -\tfrac{1}{2}k(x_f + x_i)$$

Then the work done by the spring is

$$W_s = F_{s,avg}\cos\phi\, d = F_{s,avg}\cos 0^\circ\ d$$

$$= -\tfrac{1}{2}k(x_f + x_i)(x_f - x_i) = -\tfrac{1}{2}k(x_f^2 - x_i^2)$$

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2$$

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 = U_{s,i} - U_{s,f}$$

$$U_{elastic} = U_s = \tfrac{1}{2}kx^2$$

Units of N/m m^2 = N m = J

- Total potential energy is

$$U_{total} = U_g + U_s = mgy + \tfrac{1}{2}kx^2$$

Example Problem

A block (m = 1.7 kg) and a spring (k = 310 N/m) are on a frictionless incline (θ = 30°). The spring is compressed by x_i = 0.31 m relative to its unstretched position at x = 0 and then released. What is the speed of the block when the spring is still compressed by x_f = 0.14 m?