

SP. 5  
2.

$$\begin{pmatrix} 1 & -3 & 2 \\ -2 & 1 & -4 \\ 1 & 2 & 9 \end{pmatrix} \xrightarrow[\substack{R_2 + 2R_1 \\ R_3 - R_1}]{R_2, 3} \begin{pmatrix} 1 & -3 & 2 \\ 0 & -5 & 10 \\ 0 & 5 & 2 \end{pmatrix} \xrightarrow{R_2, 3}$$

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$$\rightarrow \begin{pmatrix} 1 & -3 & 2 \\ 0 & 5 & 2 \\ 0 & -5 & 10 \end{pmatrix} \xrightarrow{R_3 + R_2} \begin{pmatrix} 1 & -3 & 2 \\ 0 & 5 & 2 \\ 0 & 0 & 12 \end{pmatrix}$$

$\Rightarrow$  the system (homogeneous) does not have any non-trivial solutions, since there is no free variable.

$$4. \begin{pmatrix} -5 & 2 & 9 \\ 1 & -2 & 6 \end{pmatrix} \xrightarrow{R_{1,2}} \begin{pmatrix} 1 & -2 & 6 \\ -5 & 2 & 9 \end{pmatrix} \xrightarrow{R_2 + 5R_1} \begin{pmatrix} 1 & -2 & 6 \\ 0 & -3 & 39 \end{pmatrix}$$

$\Rightarrow x_3$  is free  $\Rightarrow$  the system has non-trivial solutions.

$$6. \begin{pmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{pmatrix} \xrightarrow[\substack{R_2 - R_1 \\ R_3 + 3R_1}]{R_2 - R_1} \begin{pmatrix} 1 & 3 & -5 \\ 0 & 1 & -3 \\ 0 & 2 & -6 \end{pmatrix} \xrightarrow{R_3 - 2R_2} \begin{pmatrix} 1 & 3 & -5 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{R_1 - 3R_2} \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{pmatrix}$$

$x_3$  is free. We write:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -4x_3 \\ 3x_3 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix}$$

$$(9) \begin{pmatrix} 3 & -9 & 6 \\ -1 & 3 & -2 \end{pmatrix} \xrightarrow{R_1 + 3R_2} \begin{pmatrix} 0 & 0 & 0 \\ -1 & 3 & -2 \end{pmatrix}$$

P.2

We have:  $x_2, x_3$  free  
 $x_1$  basic.

$$\text{So } \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3x_2 - 2x_3 \\ x_2 \\ x_3 \end{pmatrix} = x_2 \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$$

$$11. \begin{pmatrix} 1 & -4 & -2 & 0 & 5 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{R_1 - 3R_3} \xrightarrow{R_1 + 2R_2}$$

$$\rightarrow \begin{pmatrix} 1 & -4 & 0 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow$$

$\Rightarrow x_2, x_4, x_5$  free.

$$\text{Write: } \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 4x_2 - 5x_5 \\ x_2 \\ x_5 \\ 4x_5 \\ x_5 \end{pmatrix} = x_2 \begin{pmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} -5 \\ 0 \\ 1 \\ 4 \\ 1 \end{pmatrix}$$

- 23.
- a. false: see the paragraph following eq. (3). p. 3  
The text calls  $Ax = b$  a matrix equation.
- b. True: see box before example 3.
- c. false: see the wording following Theorem 4
- e. true: see parts (c) and (a) in Theorem 4
- f. true: in Theorem 4, statement (a) is false if and only if statement (d) is false.

§ 1.2

2. Solve:  $Ax = b$  where,  $A = \begin{pmatrix} 0 & 0 & -3 \\ 0 & 5 & 4 \\ 2 & -8 & 1 \end{pmatrix} \rightarrow$

$$A = \begin{pmatrix} -3 & 0 & 0 \\ 4 & 5 & 0 \\ 1 & -8 & 2 \end{pmatrix} \xrightarrow{R_2 + R_1} \begin{pmatrix} -3 & 0 & 0 \\ 1 & 5 & 0 \\ 1 & -8 & 2 \end{pmatrix} \xrightarrow{-\frac{1}{3}R_1}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 1 & 5 & 0 \\ 1 & -8 & 2 \end{pmatrix} \xrightarrow{\substack{R_2 - R_1 \\ R_3 - R_1}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & -8 & 2 \end{pmatrix} \xrightarrow{\frac{1}{5}R_2}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -8 & 2 \end{pmatrix} \xrightarrow{R_3 + 8R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \xrightarrow{\frac{1}{2}R_3}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \text{columns of } A$$

are linearly independent  $\Rightarrow$  the vectors  
are linearly independent.