The Temperature of the Sun and Galactic Curve from the Observation of the 21-cm Line of Hydrogen

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We observe the 21-cm hyperfine line of hydrogen using a small radio telescope. From our data, we derive the brightness temperature (or effective temperature) of the sun, $T_{\mathrm{eff}\odot} = (4.81 \pm 0.19) \times 10^4$ K. We also independently verify the half-power beam width (HPBW) of the dish, HPBW $= (7.29 \pm 0.16)\,^\circ$. Further, using the varying Doppler shift of the 21-cm radiation, we calculate the galactic rotation curve of the Milky Way and qualitatively describe the general structure of our galaxy.

INTRODUCTION

The task of radio astronomy is to study extraterrestrial objects, such as the sun, utilizing their radio emission. Solar radio astronomy had a very surprising beginning. What first appeared to be a new anti-radar jamming device to the British Air Defense during World War II, was later found to be interference caused by the direct propagation of radiation from the sun. As radio waves from the sun were studied further, astronomers observed a greater flux density near 21-cm wavelengths, which, if caused by blackbody radiation alone, would require the surface temperature of the sun to be 100,000 K (94,000 K greater than the temperature of the same spectrum measured in the visible range) [1].

The quantum mechanical description of the hyperfine splitting of hydrogen's ground state (known as spin-spin coupling) has since explained this radiation. Astronomers have studied the radio sun (as the sun's radio emissions are called) extensively and derived from it many features of the sun, such as its temperature. The abundance of hydrogen in the universe and the ubiquitousness of this 21-cm radiation has also led researchers to use this line as a means of studying the dynamics of objects in the galaxy.

In this experiment, a small radio telescope is used to observe this line and deduce the temperature of the sun as well as the dynamics of galactic rotation.

THEORY

The 21-cm Line

The following discussion about the theory of the hyperfine splitting of hydrogen's ground state follows D. J. Griffiths [2]. The proton and the electron in the hydrogen atom have magnetic dipole moments given by $\vec{\mu}_p = \frac{g_p e}{2m_p}\vec{S}_e$ and $\vec{\mu}_e = \frac{-e}{m_e}\vec{S}_e$, where g_p is the gyromagnetic ratio of the proton, $\vec{S}_e$ is the spin of the electron, and $\vec{S}_p$ is the spin of the proton. These dipoles create magnetic fields according to classical electrodynamics. We may isolate the contribution to the hydrogen Hamiltonian as coming solely from the electron's Hamiltonian in the magnetic field created by the dipole moment of the proton. Therefore, we may treat the resultant correction to the hydrogen Hamiltonian as a perturbation. The first order correction to the energy is the expectation value of the perturbation in the unperturbed state,

$$E^1_{\mathrm{hf}} = \frac{\mu_0 g_p e^2}{3\pi m_p m_e a_0^3}\langle \vec{S}_p \cdot \vec{S}_e \rangle, \tag{1}$$

where μ_0 is the permeability of free space, a_0 is the Bohr radius of hydrogen. We note that if $\vec{S} = \vec{S}_e + \vec{S}_p$, we get $\vec{S}_p \cdot \vec{S}_e = (S^2 - S_e^2 - S_p^2)/2$. Since both the electron and proton have spin 1/2, $S_e^2 = S_p^2 = (3/4)\hbar^2$. Then, because the electron and proton spins can be either parallel or antiparallel, we have two cases, the former with $S^2 = \hbar^2$, and the latter with $S^2 = 0$. It follows that this spin-spin coupling splits the ground state into two levels, a triplet state, with spins parallel, and a singlet state, with spins antiparallel. Then, we may calculate the difference in energy according to equation 1, so that the frequency of a photon emitted in a transition from the triplet to the singlet state is

$$\nu = \frac{\Delta E}{h} = 1420\mathrm{MHz}, \tag{2}$$

and the corresponding wavelength is $\lambda = 21$ cm.

The Brightness Temperature of the Sun

The sun is composed of multiple layers, including the photosphere, the chromosphere, and the corona, as shown in figure 1. The photosphere emits like a blackbody corresponding to a temperature on the order of 6000 K. Over the entire extent of the chromosphere, about 2500 km, the temperature rises from $\sim$ 6000 K to $\sim$ 15,000 K. Increasing further in altitude, the corona is an extended gaseous shell around the sun with a characteristic temperature of about 10^6 K [3].

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The sun has two modes of radio emission, the quiet sun, with little or no sunspot activity and the disturbed sun, during periods of high sunspot activity. For this experiment, we are studying the brightness temperature (see below) of the quiet sun.

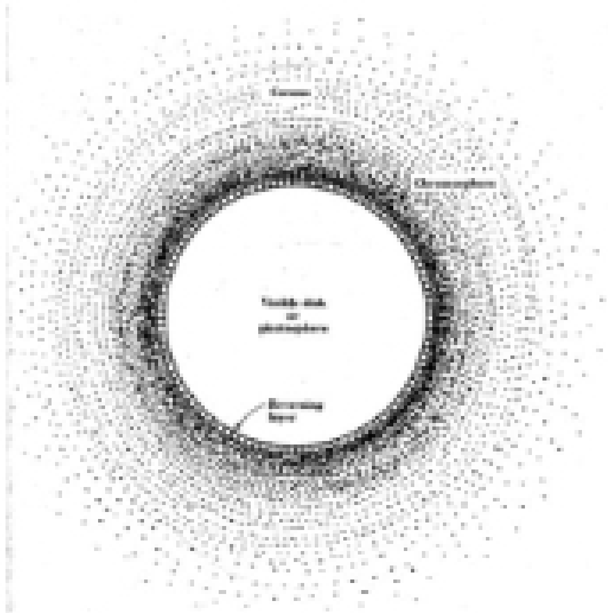

FIG. 1: The layers of the sun.

The intensity scale used in radio astronomy is antenna temperature, which is defined as the power per unit bandwidth received at the antenna terminal divided by Boltzmann's constant, k_B [4]. Expanding the power in terms of a double integral over solid angle and angular frequency, we see that [3]

$$\bar{T}^d_{\text{eff}} = \frac{1}{\Omega_s} \iint \frac{8\pi^3 c^2}{\omega^2 k_B} I(\omega, \Omega) d\omega d\Omega, \tag{3}$$

where the $I(\omega, \Omega)$ is the intensity as a function of frequency ω and solid angle Ω, and Ω_s is the solid angle of the source.

In addition, the equivalent brightness temperature of an astronomical object is defined as the physical temperature of a perfect absorber with the same angular dimensions as the object which would produce the observed antenna temperature. It can be shown that the brightness temperature,T^s_{eff} and antenna temperature are related by

$$T^s_{\text{eff}} = \bar{T}^d_{\text{eff}} \frac{\Omega_d}{\Omega_s}, \tag{4}$$

where Ω_d is the solid angle of the dish.

Fraunhofer Diffraction and Beam Width

The Fraunhofer diffraction pattern for a circular aperture and a point source is given by

$$I(\theta) = I_0 \left(\frac{2J_1(ka \sin\theta)}{ka \sin\theta} \right)^2, \tag{5}$$

where $I(\theta)$ is intensity as a function of angle, J_1 is a Bessel function of the first kind, and k is the wave number. The theoretical first minimum occurs at $\sin\theta \approx \theta = 1.22\lambda/D$. This relation is known as the Rayleigh criterion. We cannot model our data using equation 5 however due to the finite angular extent of the sun, which causes the diffraction pattern from the source sun through the small radio telescope to become more spread out and more Gaussian in nature. For this reason, all fits to angular distributions of antenna temperature were done with Gaussian curves.

EXPERIMENTAL SETUP: THE SRT

A block diagram of the experimental apparatus is shown in figure 2. The small radio telescope, or SRT, is a 7.5 ft diameter parabolic dish and receiver designed by Haystack Observatory. The beam width can be calculated using the Rayleigh criterion, BW $= 1.22 \frac{\lambda}{D} \frac{180}{\pi} \sim 7°$.

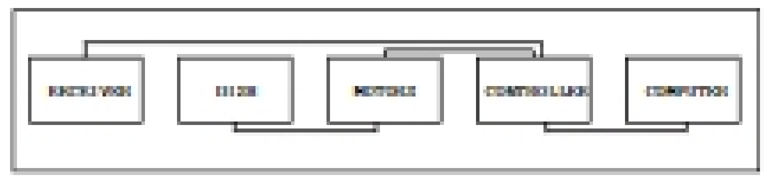

FIG. 2: Block diagram of the experimental apparatus, taken from [5].

The measurement chain proceeds as follows: the parabolic dish reflects incident radiation onto an antenna feed horn which produces the electrical signal. The signal is first pre-amplified with a gain of 24 dB, then the output passes through a 40 MHz bandpass filter that prevents out of band signals from producing intermodulation later down the measurement chain. This is then mixed with a signal produced by a local oscillator at a frequency close to the center frequency of 1420.4 MHz. The mixer forms the sum of two analog waveforms from a product. Next, the sum frequency contribution is thrown away by using another bandpass filter in the range of 0.5 - 3 MHz. This is converted into a digital signal by the analog-to-digital converter. The computer provides azimuthal and elevation tracking based on local sidereal time and allows us to track specific galactic longitudes and objects of interest in the sky [5].

Temperature calibration was done using an electronic noise diode, whose intensity and spectral distribution approximates a 115 K blackbody. The system temperature was on average 37 K, however it varied by up to 3 K day-to-day so calibration was performed in each data acquisition session.

DATA AND ERROR ANALYSIS

Data reduction and analysis was done using MATLAB and linear fit scripts provided by the Junior Lab staff.

Half-Power Beam Width

To independently calculate the half-power beam width, HPBW, we took three scans of 61 angles ($-30°$ to $30°$) about the sun in both the azimuthal and elevation dimensions. The maximum antenna temperature and corresponding index was found for each set of data and then the data was centered around $0°$. We kept only the 31 most central data points and averaged the three scans. Error bars for each data point were taken to be the standard deviation of the three corresponding points from each data set. The 31 points were fit to a Gaussian

$$T_{\rm eff}(\theta) = T_{\rm sys} + T_0 e^{(\theta-\theta_0)^2/2\sigma^2} \qquad (6)$$

Figure 3 shows the results of our Gaussian fit to this curve, which resulted in a $\chi^2_r = 0.37$. We approximated

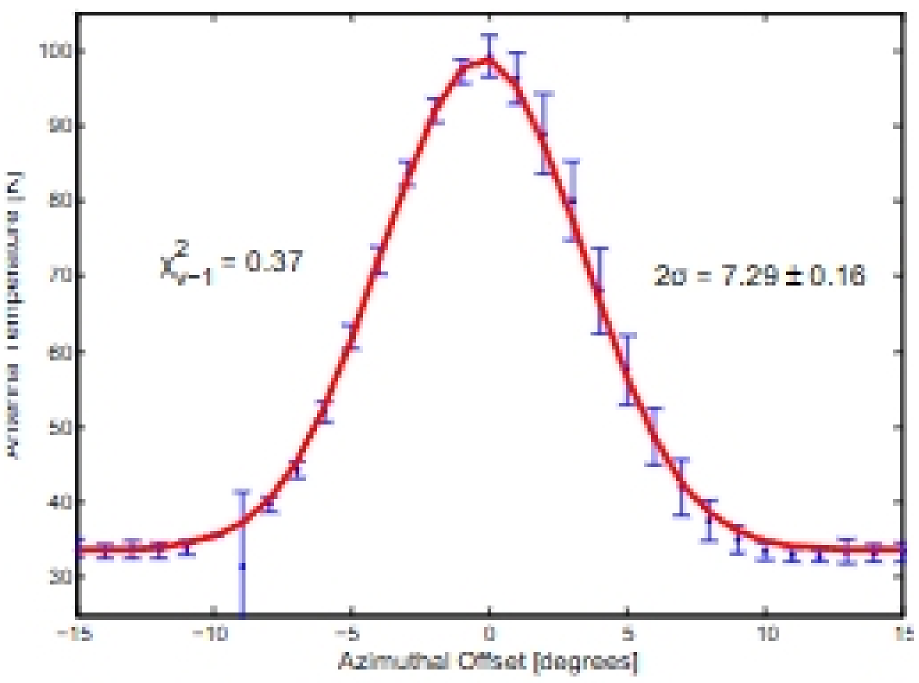

FIG. 3: Determination of half-power beam width.

the first minimum of the Bessel function, as HPBW $\approx 2\sigma = 7.29 \pm 0.16°$, which gives us reasonable agreement with the $7°$ beam width.

30 Minute Drift and n-Point Sun Scans

A drift scan of the sun consists of pointing the dish's beam at a point ahead of the sun's trajectory. The dish takes temperature measurements at this one point (without tracking anything) as the sun passes through the dish's beam. The data and the fit from the 30 minute drift scan are shown figure 4. The error on each point was taken to be 2 % of the height of the data points. The data from the drift scan was fit to equation 6 with a $\chi^2_r = 0.14$. From the extracted parameter T_0, we make a determination of the brightness temperature of the sun, $T_{\rm eff\odot}$ by using $\Omega_d = 4\pi \sin(\text{HPBW}/2) = 0.051 \pm 0.002$ sterad and $\Omega_s = (6.8 \pm 0.2) \times 10^{-5}$ sterad [1]. As our final result, we get $T_{\rm eff\odot} = (4.81 \pm 0.19) \times 10^4$ K, which is in agreement with the result of Christiansen and Warburton, who performed the same measurement in 1953 and determined $T^{\rm CW}_{\rm eff\odot} = 47000$ K [6].

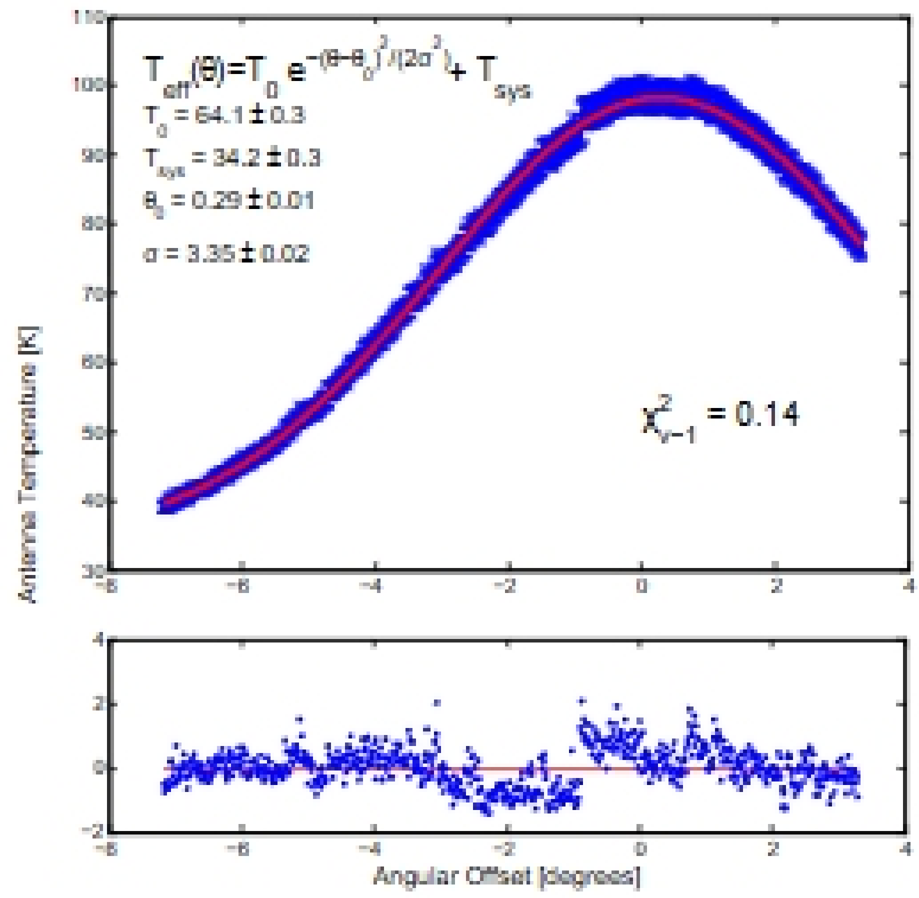

FIG. 4: Gaussian fit to a 30 minute drift scan of the sun.

An n-point scan refers to a series of measurements of the temperature along a 5×5 grid of points within a beam width on either side of the apparent position of the sun in elevation and azimuth. From this data, we were able to derive a contour plot of the brightness temperature over the photosphere of the sun, as shown in figure 5. Again, error bars are taken to be 2 % of the height of the data points.

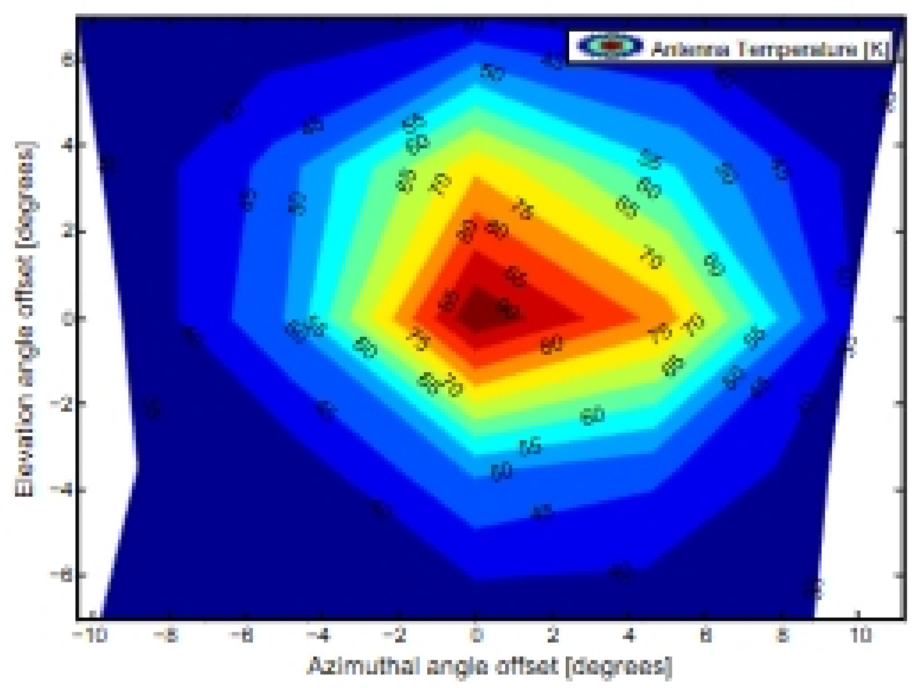

FIG. 5: Contour plot of the n-point scan over a 5×5 grid about the sun.

Galactic Rotation Curve

Forty-five measurements of the frequency spectrum were taken at a constant galactic latitude $b = 0$ (along