Lecture-8

Haralick's Edge Detector

Haralick's Edge Detector

- Fit a bi-quadratic polynomial to a small neighborhood of a pixel.
- Compute analytically second and third directional derivatives in the direction of gradient.
- If the second derivative is equal to zero, and the third derivative is negative, then that point is an edge point.

Haralick's Edge Detector

Bi-cubic polynomial:

$$f(x,y) = k_1 + k_2x + k_3y + k_4x^2 + k_5xy + k_6y^2 + k_7x^3 + k_8x^2y + k_9xy^2 + k_{10}y^3.$$

Gradient angle, defined with positive y-axis:

$$\sin\theta = \frac{k_2}{\sqrt{k_2^2 + k_3^2}},$$

$$\cos\theta = \frac{k_3}{\sqrt{k_2^2 + k_3^2}}.$$

Homework

Directional derivative $f'_\theta = \frac{\partial f}{\partial x} \cos\theta + \frac{\partial f}{\partial y} \sin\theta.$

Gradient angle, defined with positive x-axis:

Haralick's Edge Detector

$$f'_\theta = \frac{\partial f}{\partial x} \cos\theta + \frac{\partial f}{\partial y} \sin\theta.$$

$$f'_\theta(x,y) = \frac{\partial f}{\partial x}\sin\theta + \frac{\partial f}{\partial y}\cos\theta,$$

$$f''_\theta(x,y) = \frac{\partial^2 f}{\partial x^2}\sin^2\theta + \frac{\partial^2 f}{\partial y^2}\cos^2\theta + 2\frac{\partial^2 f}{\partial x \partial y}\cos\theta\sin\theta.$$

Haralick's Edge Detector

$$f(x,y) = k_1 + k_2 x + k_3 y + k_4 x^2 + k_5 xy + k_6 y^2 + k_7 x^3 + k_8 x^2 y + k_9 xy^2 + k_{10} y^3.$$

$$x = \rho \sin\theta,\ y = \rho\cos\theta$$

$$f_\theta(\rho) = C_0 + C_1\rho + C_2\rho^2 + C_3\rho^3,$$

Homework

$$\begin{aligned}
C_0 &= k_1,\\
C_1 &= k_2 \sin\theta + k_3\cos\theta,\\
C_2 &= k_4 \sin^2\theta + k_5 \sin\theta\cos\theta + k_6\cos^2\theta,\\
C_3 &= k_7 \sin^3\theta + k_8 \sin^2\theta\cos\theta + k_9 \sin\theta\cos^2\theta + k_{10}\cos^3\theta.
\end{aligned}$$

Haralick's Edge Detector

$$f_\theta(\rho) = C_0 + C_1\rho + C_2\rho^2 + C_3\rho^3,$$

$$\begin{aligned}
f_\theta'(\rho) &= C_1 + 2C_2\rho + 3C_3\rho^2,\\
f_\theta''(\rho) &= 2C_2 + 6C_3\rho,\\
f_\theta'''(\rho) &= 6C_3.
\end{aligned}$$

$f_\theta'''(\rho) < 0$, we get $6C_3 < 0$, or $C_3 < 0$.

$f_\theta''(\rho) = 2C_2 + 6C_3\rho = 0$, we get $|\frac{C_2}{3C_3}| < \rho_0$.