Chapter 5 Gravitation

Force $\vec{F}$ Field $\vec{g}$ Potential Φ

$$-\vec{\nabla}\Phi = \vec{g} = \frac{\vec{F}}{m} = \underbrace{-\frac{GM}{r^2}\vec{e}_r}_{\text{Newton}}$$

$$\boxed{\Phi(\vec{r}) = -G\int_V \frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,dv'}$$

$\Phi = -\frac{G\,\delta m}{r}$ for each small mass δm

$U = m\Phi$ potential energy

$\vec{F} = -\vec{\nabla}U$ $\vec{g} = \frac{\vec{F}}{m} = -\vec{\nabla}\frac{U}{m} = -\vec{\nabla}\Phi$

$\vec{\nabla}\cdot\vec{g} = -4\pi G\rho$ $M = \frac{-1}{4\pi G}\int_S \vec{g}\cdot d\vec{a}$ Mass enclosed in surface S.

$\nabla^2\Phi = 4\pi G\rho$ (Poisson)

($\rho = 0$ outside bodies) $\nabla^2\Phi = 0$ (Laplace)

Note: $\vec{F} = 0$ ($\Phi = \text{const}$) everywhere inside spherical shells of uniform density.

Everywhere outside spherical shells, they attract as if all mass were concentrated at their centers.

Equipotential surfaces

Lines of force

Tidal force $\leftrightarrow$ Gravitational force difference

Chapter 7 - Lagrangian Dynamics

$$L(q_i, \dot{q}_i; t) = T - U$$

$$\delta \int_{t_1}^{t_2} L\, dt = 0$$ Hamilton's Principle [Fixed endpoints]

$$\Rightarrow \boxed{\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0}$$ Euler-Lagrange Eqn

If $\frac{\partial L}{\partial q_i} = 0$, then $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = 0$

$\Rightarrow \frac{\partial L}{\partial \dot{q}_i}$ is then a constant of motion

No vectors. Scalar equations.
Focus is on the relevant generalized coordinates that best describe the configuration of the system.
Powerful problem-solving formalism.

Hamiltonian Dynamics

$$p_i \equiv \frac{\partial L}{\partial \dot{q}_i} \longrightarrow \dot{q}_i(q_i, p_i)$$

$$H(q_i, p_i; t) = \sum_i p_i \dot{q}_i(q_i, p_i) - L(q_i, \dot{q}_i(q_i, p_i); t)$$

$$\boxed{\begin{aligned} \dot{q}_i &= \frac{\partial H}{\partial p_i} \\ \dot{p}_i &= -\frac{\partial H}{\partial q_i} \end{aligned}}$$

Chapter 8 Overview

2 objects with central force, separation $\vec{r}$
Reduced mass μ, 1-body problem in 2D.

$$\theta(r) = \int \frac{\pm (\ell/r^2)\, dr}{\sqrt{2\mu\left(E - U - \frac{\ell^2}{2\mu r^2}\right)}}$$

Gravitation: $U = -\frac{k}{r}$ $\quad (k = GMm)$

$$r = \frac{\alpha}{1 + \varepsilon \cos\theta} \quad \text{(conic sections)}$$

$$\alpha = \frac{\ell^2}{\mu k} \qquad \varepsilon = \sqrt{1 + \frac{2E\ell^2}{\mu k^2}}$$

Effective potential $V(r) = U(r) + \frac{\ell^2}{2\mu r^2}$
Turning points "apsides"

Kepler's laws, $\frac{dA}{dt} = \frac{\ell}{2\mu}$, $\tau^2 = \frac{4\pi^2\mu}{k} a^3$

Planetary toolkit.
Hohmann transfers
Gravity assists