PHYS 1444 – Section 004
Lecture #21

Monday, April 30, 2007
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- Maxwell's Equations
- Production of Electromagnetic Waves
- EM Waves from Maxwell's Equations
- Speed of EM Waves

Example 32 – 1

Charging capacitor. A 30-pF air-gap capacitor has circular plates of area A=100cm^2. It is charged by a 70-V battery through a 2.0-Ω resistor. At the instant the battery is connected, the electric field between the plates is changing most rapidly. At this instance, calculate (a) the current into the plates, and (b) the rate of change of electric field between the plates. (c) Determine the magnetic field induced between the plates. Assume E is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.

Since this is an RC circuit, the charge on the plates is: $Q = CV_0\left(1 - e^{-t/RC}\right)$

For the initial current (t=0), we differentiate the charge with respect to time.

$$I_0 = \left.\frac{dQ}{dt}\right|_{t=0} = \left.\frac{CV_0}{RC} e^{-t/RC}\right|_{t=0} = \frac{V_0}{R} = \frac{70V}{2.0\Omega\cdot} = 35A$$

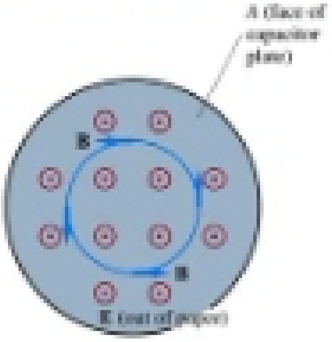
The electric field is $E = \frac{\sigma}{\varepsilon_0} = \frac{Q/A}{\varepsilon_0}$

Change of the electric field is $\frac{dE}{dt} = \frac{dQ/dt}{A\varepsilon_0} = \frac{35A}{\left(8.85\times10^{-12}\ C^2/N\cdot m^2\right)\cdot\left(1.0\times10^{-2}\ m^2\right)} = 4.0\times10^{14}\ V/m\cdot s$

Example 32 – 1

(c) Determine the magnetic field induced between the plates. Assume **E** is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.

The magnetic field lines generated by changing electric field is perpendicular to E and is circular due to symmetry



Whose law can we use to determine B?

Extended Ampere's Law w/ I_{encl}=0!

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

We choose a circular path of radius r, centered at the center of the plane, following the B.

For $r<r_{plate}$, the electric flux is $\Phi_E = EA = E\pi r^2$ since E is uniform throughout the plate

So from Ampere's law, we obtain $B \cdot (2\pi r) = \mu_0 \varepsilon_0 \frac{d\left(E\pi r^2\right)}{dt} = \mu_0 \varepsilon_0 \pi r^2 \frac{dE}{dt}$

Solving for B $\Rightarrow$ $B = \mu_0 \varepsilon_0 \frac{r}{2} \frac{dE}{dt}$ For $r<r_{plate}$

Since we assume E=0 for $r>r_{plate}$, the electric flux beyond the plate is fully contained inside the surface. $\Phi_E = EA = E\pi r_{plate}^2$

So from Ampere's law, we obtain $B \cdot (2\pi r) = \mu_0 \varepsilon_0 \frac{d\left(E\pi r_{plate}^2\right)}{dt} = \mu_0 \varepsilon_0 \pi r_{plate}^2 \frac{dE}{dt}$

Solving for B $\Rightarrow$ $B = \frac{\mu_0 \varepsilon_0 r_{plate}^2}{2r} \frac{dE}{dt}$ For $r>r_{plate}$