CSE 321 Discrete Structures

Winter 2008
Lecture 25
Graph Theory

Graph Theory

- Graph formalism
  - G = (V, E)
  - Vertices
  - Edges
- Directed Graph
  - Edges ordered pairs
- Undirected Graph
  - Edges sets of size two

Graph examples

- Communication Networks
- Road networks

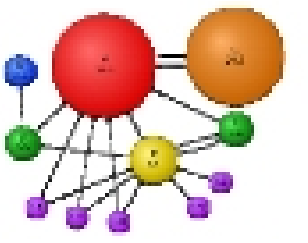
Social networks

- Community Graph
  - Linked In, Face Book
- Transactions
  - Ebay
- Authorship
  - Erdos Number

The web graph

Page Rank

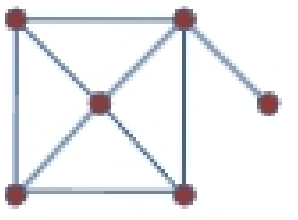
- Determine the value of a page based on link analysis
- Model of randomly traversing a graph
  - Weighting factors on nodes
  - Damping (random transitions)



Graph terminology

- Neighborhood
- Degree

Degree sequence



- Find a graph with degree sequence
  - 3, 3, 2, 1, 1
- Find a graph with degree sequence
  - 3, 3, 3, 3, 3

Handshake Theorem

$$2e = \sum_{v \in V} \deg(v)$$

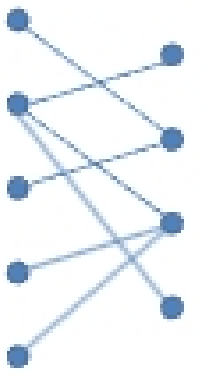
Directed Degree Theorem

$$\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |E|$$

Special Graphs

- Complete Graphs K_n
- Cycle C_n
- Hypercube Q_n
- Mesh $M_{n,m}$

Bipartite Graphs



2-coloring

- A graph is two colorable iff all cycles have even length

Graph Representations

- Adjacency Lists
- Adjacency Matrices
- Incidence Matrices

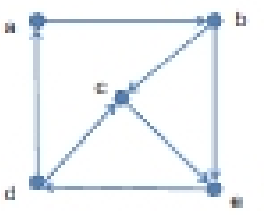
Graph Connectivity

Strong connectivity vs. Weak Connectivity

Strongly Connected Components

Counting Paths

Let A be the Adjacency Matrix. What is A^2?



$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$