8-12

a) If the process uses 66.7% of the specification band, then $6\sigma = 0.667(USL-LSL)$ then assume $\bar{\bar{x}} = \mu$ since the process is centered

$3\sigma=0.667\,(USL-\bar{\bar{x}})=0.667(\bar{\bar{x}} - LSL)=0.667(USL-\mu)$

$4.5\sigma=USL-\mu=LSL-\mu$

$C_{pk}=\min[\frac{4.5\sigma}{3\sigma},\frac{4.5\sigma}{3\sigma}]=1.5$

Since C_p and C_{pk} exceeds unity, the natural tolerance limits lie inside the specification limits and very few defective units will be produced.

b) Assuming a normal distribution with $6\sigma = 0.667(USL – LSL)$ and a centered process, then

$3\sigma = 0.667(USL - \mu)$. Consequently, $USL - \mu = 4.5\sigma$ and $\mu - LSL = 4.5\sigma$

$$\begin{aligned} P(X > USL) &= P\left(Z > \frac{4.5\sigma}{\sigma}\right) \\ &= P(Z > 4.5) \\ &= 1 - P(Z < 4.5) \\ &= 1 - 1 \\ &= 0 \end{aligned}$$

By symmetry, the fraction defective is $2[P(X > USL)] = 0$.

8-13

a) $\hat{\sigma} = \frac{\bar{r}}{d_2} = \frac{4.947}{1.693} = 2.922$ or $\hat{\sigma} = 0.0002922$

$$C_p = \frac{USL - LSL}{6(\hat{\sigma})} = \frac{0.4040 - 0.4020}{6(0.0002922)} = 1.141$$

$$\begin{aligned} C_{pk} &= \min\left[\frac{USL - \bar{\bar{x}}}{3\hat{\sigma}}, \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}}\right] \\ &= \min\left[\frac{0.4040 - 0.403428}{3(0.0002922)}, \frac{0.4030428 - 0.4020}{3(0.0002922)}\right] \\ &= \min[0.6525, 1.629] \\ &= 0.6525 \end{aligned}$$

Since C_p exceeds unity, the natural tolerance limits lie inside the specification limits and very few defective units will be produced.

Since Cpk ≠ Cp the process is off center.

b) Assuming a normal distribution with $\hat{\mu} = 0.403248$ and $\hat{\sigma} = 0.0002922$

$$\begin{aligned} P(X < LSL) &= P\left(Z < \frac{LSL - \hat{\mu}}{\hat{\sigma}}\right) \\ &= P(Z < -5.07) \\ &= 0 \end{aligned}$$

$$\begin{aligned} P(X > USL) &= P\left(Z > \frac{USL - \hat{\mu}}{\hat{\sigma}}\right) \\ &= P(Z > 1.78) \\ &= 1 - P(Z < 1.78) \\ &= 1 - 0.9625 \\ &= 0.0375 \end{aligned}$$

Therefore, the proportion nonconforming is given by

$$\begin{aligned} P(X<LSL) + P(X>USL) &= 0.0375 + 0 \\ &= 0.0375 \end{aligned}$$

8-14

a) Assuming a normal distribution with $\hat{\mu} = 18.925$ and $\hat{\sigma} = \frac{\bar{r}}{d_2} = \frac{0.39}{2.059} = 0.189$

$$P(X < LSL) = P\left(Z < \frac{LSL - \hat{\mu}}{\hat{\sigma}}\right)$$
$$= P\left(Z < \frac{18.00 - 18.925}{0.189}\right)$$
$$= P(Z < -4.89)$$
$$= 0$$

$$P(X > USL) = P\left(Z > \frac{USL - \hat{\mu}}{\hat{\sigma}}\right)$$
$$= P\left(Z > \frac{19.00 - 18.925}{0.189}\right)$$
$$= P(Z > 0.40)$$
$$= 1 - P(Z < 0.40)$$
$$= 1 - 0.65542$$
$$= 0.34458$$

Therefore, the proportion nonconforming is given by

$P(X<LSL) + P(X>USL) = 0 + 0.3446$

$= 0.3446$

b)

$$Cp = \frac{USL - LSL}{6(\hat{\sigma})} = \frac{19.00 - 18.00}{6(0.189)} = 0.882$$

$$Cpk = \min\left[\frac{USL - \bar{\bar{x}}}{3\hat{\sigma}}, \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}}\right]$$
$$= \min\left[\frac{19.00 - 18.925}{3(0.189)}, \frac{18.925 - 18.00}{3(0.189)}\right]$$
$$= \min[0.132, 1.63]$$
$$= 0.132$$

Since C_p is less than unity, many defective units are being produced.

$C_{pk} \neq C_p$ the process is not centered.

8-16

a) If the process uses 85% of the spec band then $6\sigma = 0.85(USL - LSL)$ and

$$Cp = \frac{USL - LSL}{0.85(USL - LSL)} = \frac{1}{0.85} = 1.18$$

Assume $\bar{\bar{x}} = \mu$ and $3\sigma = 0.85(USL - \bar{\bar{x}}) = 0.85(\mu - LSL)$

Therefore,

$$Cpk = \min\left[\frac{3.53\hat{\sigma}}{3\hat{\sigma}}, \frac{3.53\hat{\sigma}}{3\hat{\sigma}}\right] = 1.18$$

Since C_p and C_{pk} exceed unity, the natural tolerance limits lie inside the specification limits and very few defective units will be produced.

b) Assuming a normal distribution with $6\sigma = 0.85(USL - LSL)$ and a centered process, then $3\sigma = 0.85(USL - \mu)$. Consequently, $USL - \mu = 3.53\sigma$ and $\mu - LSL = 3.53\sigma$

$$P(X > USL) = P\left(Z > \frac{3.53\sigma}{\sigma}\right)$$
$$= P(Z > 3.53)$$
$$= 1 - P(Z < 3.53)$$
$$= 1 - 0.9998$$
$$= 0.0002$$

By symmetry, the fraction defective is 2[P(X > USL)] = 0.0004.

8-17

Assuming a normal distribution with $\hat{\mu} = 306.28$ and $\hat{\sigma} = 22.923$

$$P(X < LSL) = P\left(Z < \frac{LSL - \hat{\mu}}{\hat{\sigma}}\right)$$
$$= P\left(Z < \frac{260 - 306.28}{22.923}\right)$$
$$= P(Z < -2.02)$$
$$= 0.0217$$

$$P(X > USL) = P\left(Z > \frac{USL - \hat{\mu}}{\hat{\sigma}}\right)$$
$$= P\left(Z > \frac{340 - 306.28}{22.923}\right)$$
$$= P(Z > 1.47)$$
$$= 1 - P(Z < 1.47)$$
$$= 1 - 0.9292$$
$$= 0.0708$$

Therefore, the proportion nonconforming is given by

$$P(X<LSL) + P(X>USL) = 0.0217 + 0.0708$$
$$= 0.0925$$

$$Cp = \frac{USL - LSL}{6(\hat{\sigma})} = \frac{340 - 260}{6(22.923)} = 0.582$$

$$Cpk = \min\left[\frac{USL - \bar{\bar{x}}}{3\hat{\sigma}}, \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}}\right]$$
$$= \min\left[\frac{340 - 306.28}{3(22.923)}, \frac{306.28 - 260}{3(22.923)}\right]$$
$$= \min[0.490, 0.673]$$
$$= 0.490$$

The process capability is marginal.

8-23

a) The control limits are

UCL = 1.676

CL = 0.62

LCL = 0