CSE 321 Discrete Structures

Winter 2008

Lecture 5

Rules of Inference

Announcements

- Reading for this week
  - Today: 1.5
  - Friday: 1.6
- Homework 2
  - Due Wednesday, January 23
- January 21, No class

Valid Arguments

- Classical logic
- Artificial Intelligence: Automated Reasoning

Reasoning

- "If Seattle won last Saturday they would be in the playoffs"
- "Seattle is not in the playoffs"
- Therefore . . .

Proofs

- Start with hypotheses and facts
- Use rules of inference to extend set of facts
- Result is proved when it is included in the set

Rules of Inference

$$\frac{p \quad p \rightarrow q}{\therefore q} \qquad \frac{\neg q \quad p \rightarrow q}{\therefore \neg p} \qquad \frac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r} \qquad \frac{p \vee q \quad \neg p}{\therefore q}$$

$$\frac{p}{\therefore p \vee q} \qquad \frac{q}{\therefore p \vee q} \qquad \frac{p \wedge q}{\therefore p} \qquad \frac{p \vee q \quad \neg p \vee r}{\therefore q \vee r}$$

$$\frac{\forall x P(x)}{\therefore P(c)} \qquad \frac{P(c) \text{ for any } c}{\therefore \forall x P(x)} \qquad \frac{\exists x P(x)}{\therefore P(c) \text{ for some } c} \qquad \frac{P(c) \text{ for some } c}{\therefore \exists x P(x)}$$

Example 6

- Hypotheses:
  - It is not sunny this afternoon and it is colder than yesterday
  - We will go swimming only if it is sunny
  - If we do not go swimming, then we will take a canoe trip
  - If we take a canoe trip, we will be home by sunset
- Show:
  - We will be home by sunset

Classical Logic

- All men are mortal
- Socrates is a man
- Therefore, Socrates is mortal

Example 13

- Show "A student in this class has not read the book", and "Everyone in this class passed the exam" imply "Someone who passed the exam has not read the book"

C(x): *x* is in the class
B(x): *x* has read the book
P(x): *x* passed the exam

Proofs

- Proof methods
  - Direct proof
  - Contrapositive proof
  - Proof by contradiction
  - Proof by equivalence

Direct Proof

- If n is odd, then n^2 is odd

Definition
n is even if $n = 2k$ for some integer *k*
n is odd if $n = 2k+1$ for some integer *k*

Contrapositive

- Sometimes it is easier to prove $\neg q \rightarrow \neg p$ than it is to prove $p \rightarrow q$
- Prove that if $ab \leq n$ then $a \leq n^{1/2}$ or $b \leq n^{1/2}$

Proof by contradiction

- Suppose we want to prove p is true.
- Assume p is false, and derive a contradiction

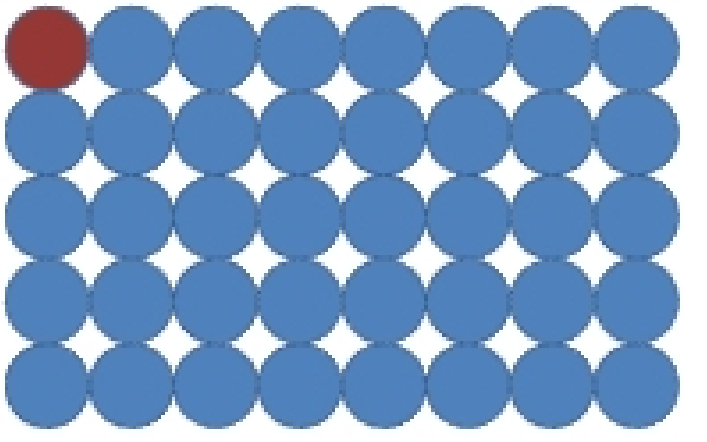
Contradiction example

- Show that at least four of any 22 days must fall on the same day of the week

Equivalence Proof

- To show $p_1 \leftrightarrow p_2 \leftrightarrow p_3$, we show $p_1 \rightarrow p_2$, $p_2 \rightarrow p_3$, and $p_3 \rightarrow p_1$
- Show that the following are equivalent
  - p_1: n is even
  - p_2: n-1 is odd
  - p_3: n^2 is even

The Game of Chomp



Theorem: The first player can always win in an $n \times m$ game

- Every position is a forced win for player A or player B (this fact will be used without proof)
- Any finite length, deterministic game with no ties is a win for player A or player B under optimal play

Proof

- Consider taking the lower right cell
  - If this is a forced win for A, then done
  - Otherwise, B has a move m that is a forced win for B, so if A started with this move, A would have a forced win