MAT 2705-03/04 11F Takehome Test 3 Answers (1)

① a) $[9x'' + 6x' + 37x = F(t)]/9 \to$

$$x'' + \underbrace{\tfrac{2}{3}}_{k_0} x' + \underbrace{\tfrac{37}{9}}_{\omega_0^2} x = \tfrac{1}{9} F(t)$$

$$\omega_0 = \sqrt{\tfrac{37}{9}} = \tfrac{\sqrt{37}}{3} \approx 2.028$$

$k_0 = 2/3 \approx 0.667$ $\qquad Q = \omega_0 \tau_0 = \frac{\sqrt{37}}{2} \approx 3.041$

$\tau_0 = 1/k_0 = 3/2 = 1.5$ $\qquad T_0 = \frac{2\pi}{\omega_0} = \frac{6\pi}{\sqrt{37}} \approx 3.099$

b) $x \sim e^{rt}$: $9r^2 e^{rt} + 6r e^{rt} + 37 e^{rt} = 0$

$$9r^2 + 6r + 37 = 0, \quad r = \frac{-6 \pm \sqrt{36 - 4\cdot 9 \cdot 37}}{2\cdot 9} = \frac{-6 \pm 36i}{2\cdot 9} = -\tfrac{1}{3} \pm 2i$$

$e^{rt} = e^{-\frac{t}{3}} e^{\pm 2i} = e^{-\frac{t}{3}}(\cos 2t \pm i \sin 2t)$ complex basis $\to$

$\to e^{-t/3}\cos 2t,\ e^{-t/3}\sin 2t$ real basis of soln space

Gen soln: $\boxed{x = e^{-t/3}(c_1 \cos 2t + c_2 \sin 2t)}$

$x' = -\frac{1}{3} e^{-t/3}(c_1 \cos 2t + c_2 \sin 2t) + e^{-t/3}(-2c_1 \sin 2t + 2c_2 \cos 2t)$

$k_1 = \frac{1}{3} \approx 0.333$, $\omega_1 = 2$

$\tau_1 = 1/k_1 = 3$ $\qquad T_1 = 2\pi/\omega_1 = \pi \approx 3.142$

c) $9x'' + 6x' + 37x = 5\cos 2t$

$37\ [x_p = c_3 \cos 2t + c_4 \sin 2t]$

$6\ [x_p' = 2c_3 \sin 2t + 2c_4 \cos 2t]$

$9\ [x_p'' = -4c_3 \cos 2t - 4c_4 \sin 2t]$

$9x_p'' + 6x_p' + 37x_p = [(37-36)c_3 + 12c_4]\cos 2t = 5\cos 2t$
$\qquad + [-12c_3 + (37-36)c_4]\sin 2t$

$c_3 + 12c_4 = 5$
$-12c_3 + c_4 = 0$
$$\begin{bmatrix} 1 & 12 \\ -12 & 1 \end{bmatrix}\begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \frac{1}{145}\begin{bmatrix} 1 & -12 \\ 12 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \end{bmatrix} = \frac{5}{5\cdot 29}\begin{bmatrix} 1 \\ 12 \end{bmatrix} = \begin{bmatrix} 1/29 \\ 12/29 \end{bmatrix}$$

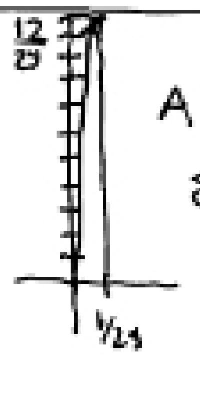
$x = x_h + x_p = e^{-t/3}(c_1 \cos 2t + c_2 \sin 2t) + \frac{1}{29}[\cos 2t + 12 \sin 2t]$

$x' = -\frac{1}{3} e^{-\frac{t}{3}}(c_1 \cos 2t + c_2 \sin 2t) + \frac{1}{29}[-2\sin 2t + 24 \cos 2t]$
$\qquad + e^{-t/3}(-2c_1 \sin 2t + 2c_2 \cos 2t)$

$x(0) = c_1 + 1/29 = 0 \to c_1 = -1/29$

$x'(0) = -\frac{c_1}{3} + 2c_2 + 24 = 0 \to c_2 = \underbrace{\tfrac{1}{3}\left(-\tfrac{1}{29}\right) - \left(\tfrac{24}{29}\right)}_{2} = \frac{-73}{174}$

$$\boxed{x = e^{-t/3}\left(\tfrac{-73}{174}\sin 2t - \tfrac{1}{29}\cos 2t\right) + \underbrace{\tfrac{1}{29}[\cos 2t + 12 \sin 2t]}_{x_{ss}}}$$




$A = \frac{1}{29}\sqrt{1+12^2} = \frac{\sqrt{145}}{29} = \frac{\sqrt{5\cdot 29}}{29} = \sqrt{\frac{5}{29}} \approx 0.415$

$\delta = \arctan 12 \approx 1.488 \approx 85.2° \approx 0.237$ cycles

so $x_{ss} = \sqrt{\frac{5}{29}}\cos(2t - \arctan 12)$

d) $9x'' + 6x' + 37x = F_0 \cos \omega t$

$37\ [x_p = c_3 \cos \omega t + c_4 \sin \omega t]$

$6\ [x_p' = -\omega c_3 \sin \omega t + \omega c_4 \cos \omega t]$

$9\ [x_p'' = -\omega^2 c_3 \cos \omega t - \omega^2 c_4 \sin \omega t]$

$9x_p'' + 6x_p' + 37x_p = [(37-9\omega^2)c_3 + 6\omega c_4]\cos \omega t$
$\qquad + [-6\omega c_3 + (37-9\omega^2)c_4]\sin \omega t$
$\qquad = F_0 \cos \omega t$

$$\begin{bmatrix} 37-9\omega^2 & 6\omega \\ -6\omega & 37-9\omega^2 \end{bmatrix}\begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \frac{1}{(37-9\omega^2)^2 + 36\omega^2}\begin{bmatrix} 37-9\omega^2 & -6\omega \\ 6\omega & 37-9\omega^2 \end{bmatrix}\begin{bmatrix} F_0 \\ 0 \end{bmatrix} = \frac{F_0}{(37-9\omega^2)^2 + 36\omega^2}\begin{bmatrix} 37-9\omega^2 \\ 6\omega \end{bmatrix}$$

$$\boxed{x_{ss} = \frac{F_0}{(37-9\omega^2)^2 + 36\omega^2}\left[(37-9\omega^2)\cos \omega t + 6\omega \sin \omega t\right]}$$

e) $$\boxed{A(\omega)} = F_0 \frac{\sqrt{(37-9\omega^2)^2 + 36\omega^2}}{(37-9\omega^2)^2 + 36\omega^2} = \boxed{\frac{F_0}{\sqrt{(37-9\omega^2)^2 + 36\omega^2}}}$$
$$= F_0(1369 - 630\omega^2 + 81\omega^4)^{-1/2}$$

$A(2) = \frac{5}{\sqrt{(37-36)^2 + 36\cdot 4}} = \frac{5}{\sqrt{144+1}} = \frac{5}{\sqrt{145}} = \frac{5\sqrt{145}}{(145)}$

$= \frac{\sqrt{145}}{29}$ ✓ $\qquad A(0) = F_0/37$

$0 = A'(\omega) = F_0\left(-\frac{1}{2}\right)(\cdots)^{-3/2}\left(\underbrace{2(37-9\omega^2)(-18\omega) + 72\omega}\right)$

$0 = 36\omega(-(37-9\omega^2) + 2) = 36\omega(9\omega^2 - 35)$

$\omega = 0$, $\boxed{\omega_p = \sqrt{35/9} = \sqrt{35}/3 \approx 1.972}$

$A(\omega_p) = F_0\left((37-35)^2 + 36\cdot(35/9)\right)^{-1/2}$
$= F_0(4 + 4(35))^{-1/2} = F_0(4\cdot 36)^{-1/2} = F_0/12$

$\boxed{A(\omega_p)/F_0 = 1/12 \approx 0.0833}$

$F_0 = 5$: $A(2) = 5\left((37-9\cdot 4)^2 + 36\cdot 4\right)^{-1/2}$
$= 5(145)^{-1/2} = \frac{5}{\sqrt{5\cdot 29}} = \sqrt{\frac{5}{29}}$ ✓

$\frac{A(\omega_p)}{A(0)} = \frac{F_0/12}{F_0/37} = \boxed{\frac{37}{12} \approx 3.083}$ ← pretty close!
$Q \approx 3.041$

MAT 2705-03/04 11F Takehome Test 3 Answers (2)

② a) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}' = \underbrace{\begin{bmatrix} -1/2 & 0 & 1/2 \\ 1/2 & -1/5 & 0 \\ 0 & 1/5 & -1/2 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{bmatrix} = \begin{bmatrix} 18 \\ 0 \\ 0 \end{bmatrix}$

b) $0 = |A - \lambda I| = \begin{vmatrix} -\frac12-\lambda & 0 & 1/2 \\ 1/2 & -1/5-\lambda & 0 \\ 0 & 1/5 & -1/2-\lambda \end{vmatrix} = -\lambda^3 - \frac{6}{5}\lambda^2 - \frac{9}{20}\lambda$
$= -\lambda(\lambda^2 + \frac{6}{5}\lambda + \frac{9}{20})$

$\to \lambda = 0, \ -\frac{3}{5} \pm \frac{3}{10} i$

$\lambda_1 = \lambda = 0$: $A = \begin{bmatrix} -1/2 & 0 & 1/2 \\ 1/2 & -1/5 & 0 \\ 0 & 1/5 & -1/2 \end{bmatrix} \xrightarrow{\text{rref}} \begin{array}{c} \begin{array}{ccc} L & L & F \end{array} \\ \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -5/2 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x_3 = t, \ x_1 = +t, \ x_2 = +5/2\, t$

$\langle x_1, x_2, x_3 \rangle = \langle t, 5/2\, t, t \rangle = t \underbrace{\langle 1, 5/2, 1 \rangle}_{\vec{b}_1}$

$\lambda_2 = \lambda = -\frac{3}{5} + \frac{3}{10} i$: $A - \lambda I = \begin{bmatrix} -\frac12 + \frac35 - \frac{3}{10} i & 0 & 1/2 \\ 1/2 & -\frac15 + \frac35 - \frac{3}{10} i & 0 \\ 0 & 1/5 & -\frac12 + \frac35 - \frac{3}{10} i \end{bmatrix}$

$\xrightarrow{\text{rref}} \begin{array}{c} \begin{array}{ccc} L & L & F \end{array} \\ \begin{bmatrix} 1 & 0 & \frac12(1+3i) \\ 0 & 1 & \frac12(1-3i) \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x_3 = t, \ x_1 = -\frac12(1+3i)t, \ x_2 = -\frac12(1-3i)t$

$\langle x_1, x_2, x_3 \rangle = \langle t, -\frac12(1+3i)t, -\frac12(1-3i)t \rangle$
$= t \underbrace{\langle 1, -\frac12(1+3i), -\frac12(1-3i) \rangle}$

$\vec{b}_2 = \overline{\vec{b}_3} \qquad \lambda_3 = \overline{\lambda_2}$

$B = \langle \vec{b}_1 | \vec{b}_2 | \vec{b}_3 \rangle = \begin{bmatrix} 1 & -\frac12(1+3i) & -\frac12(1-3i) \\ 5/2 & -\frac12(1-3i) & -\frac12(1+3i) \\ 1 & 1 & 1 \end{bmatrix}$

$(\vec{x} = B\vec{y}, \ \vec{y} = B^{-1}\vec{x}) \to \vec{x}' = A\vec{x} \to \vec{y}' = A_B \vec{y}$

$A_B = B^{-1} A B = \begin{bmatrix} -1/2 & 0 & 0 \\ 0 & -1/5 & 0 \\ 0 & 0 & -1/2 \end{bmatrix}$

$\begin{bmatrix} y_1' \\ y_2' \\ y_3' \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0\, y_1 \\ (-\frac35 + \frac{3}{10} i)\, y_2 \\ (-\frac35 - \frac{3}{10} i)\, y_3 \end{bmatrix}$

$y_1 = c_1$
$y_2 = c_2 e^{-\frac35 t} e^{\frac{3}{10} i t}$
$y_3 = c_3 e^{-\frac35 t} e^{-\frac{3}{10} i t}$

$\vec{x} = c_1 \vec{b}_1 + c_2 \left( e^{-3t/5} e^{3it/10} \vec{b}_2 \right) + \text{c.c.}$

find Re, Im parts for real basis

② b) $e^{-\frac{3t}{5}} \left(\cos\frac{3t}{10} + i \sin\frac{3t}{10}\right) \begin{bmatrix} -\frac12(1+3i) \\ -\frac12(1-3i) \\ 1 \end{bmatrix}$

$= e^{-\frac{3t}{5}} \begin{bmatrix} -\frac12\left(\cos\frac{3t}{10} - 3\sin\frac{3t}{10} + 3i\cos\frac{3t}{10} + i\sin\frac{3t}{10}\right) \\ -\frac12\left(\cos\frac{3t}{10} + 3\sin\frac{3t}{10} - 3i\cos\frac{3t}{10} + i\sin\frac{3t}{10}\right) \\ \cos\frac{3t}{10} + i\sin\frac{3t}{10} \end{bmatrix}$

$= \underbrace{e^{-\frac{3t}{5}} \begin{bmatrix} -\frac12(\cos\frac{3t}{10} - 3\sin\frac{3t}{10}) \\ -\frac12(\cos\frac{3t}{10} + 3\sin\frac{3t}{10}) \\ \cos\frac{3t}{10} \end{bmatrix}}_{\vec{X}_1} + i\, \underbrace{e^{-\frac{3t}{5}} \begin{bmatrix} -\frac12(3\cos\frac{3t}{10} + \sin\frac{3t}{10}) \\ -\frac12(-3\cos\frac{3t}{10} + \sin\frac{3t}{10}) \\ \sin\frac{3t}{10} \end{bmatrix}}_{\vec{X}_2}$

$\{\vec{X}_1, \vec{X}_2\}$ real basis of this subspace of soln space so:

$\vec{x} = c_1 \begin{bmatrix} 1 \\ 5/2 \\ 1 \end{bmatrix} + c_2 \vec{X}_1 + c_3 \vec{X}_2$

$\begin{bmatrix} 18 \\ 0 \\ 0 \end{bmatrix} = \vec{x}(0) = c_1 \begin{bmatrix} 1 \\ 5/2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -1/2 \\ -1/2 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} -\frac32 \\ 3/2 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1/2 & -3/2 \\ 5/2 & -1/2 & 3/2 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \left[ \ \downarrow \ \right]^{-1} \begin{bmatrix} 18 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \\ -8 \end{bmatrix}$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 5/2 \\ 1 \end{bmatrix} - 4 e^{-\frac{3t}{5}} \begin{bmatrix} -\frac12(c - 3s) \\ -\frac12(c + 3s) \\ c \end{bmatrix} - 8 e^{-\frac{3t}{5}} \begin{bmatrix} -\frac12(3c + s) \\ -\frac12(-3c + s) \\ s \end{bmatrix}$

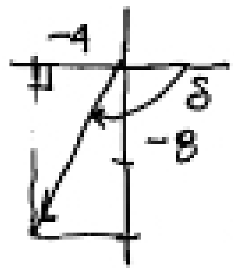
$= \begin{bmatrix} 4 + e^{-\frac{3t}{5}}(2c - 6s + 12c + 4s) \\ 10 + e^{-\frac{3t}{5}}(2c + 6s - 12c + 4s) \\ 4 - 4e^{-\frac{3t}{5}}(c + 2s) \end{bmatrix}$

$= \boxed{\begin{bmatrix} 4 + e^{-\frac{3t}{5}}\left(14\cos\frac{3t}{10} - 2\sin\frac{3t}{10}\right) \\ 10 + e^{-\frac{3t}{5}}\left(-10\cos\frac{3t}{10} + 10\sin\frac{3t}{10}\right) \\ 4 - 4e^{-\frac{3t}{5}}\left(\cos\frac{3t}{10} + 2\sin\frac{3t}{10}\right) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}$

agrees with Maple!

c) $\boxed{\vec{x}_\infty = \begin{bmatrix} 4 \\ 10 \\ 4 \end{bmatrix}}$ since $e^{-\frac{3t}{5}} \to 0$ as $t \to \infty$

d) $x_3 - x_{3\infty} = -4e^{-\frac{3t}{5}}\left(\cos\frac{3t}{10} + 2\sin\frac{3t}{10}\right)$




$A = 4\sqrt{1+2^2} = 4\sqrt5 \approx 8.944$

$\delta = -\pi + \arctan 2 \approx -0.324$ cycles
$\approx -2.034 \qquad \approx -116.6°$

$\tau = \frac53 \quad 5\tau = \frac{25}{3} \approx 8\frac13 \approx 10$ for viewing window

MAT 2705-03/04 11F TakehomeTest 3 Answers (3)

3 a) $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \underbrace{\begin{bmatrix} -3 & 4 \\ 6 & -5 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$

$$0 = |A - \lambda I| = \begin{vmatrix} -3-\lambda & 4 \\ 6 & -5-\lambda \end{vmatrix} = (\lambda+3)(\lambda+5) - 24 = \lambda^2 + 8\lambda + 15 - 24 = \lambda^2 + 8\lambda - 9$$

$\lambda = 1, -9 \qquad \lambda_1 = 1, \lambda_2 = -9$

$$\lambda = 1: \ A - I = \begin{bmatrix} -3-1 & 4 \\ 6 & -5-1 \end{bmatrix} = \begin{bmatrix} -4 & 4 \\ 6 & -6 \end{bmatrix} \to \begin{array}{c} \begin{array}{cc} L & F \end{array} \\ \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_2 = t \quad x_1 = t \qquad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} t \\ t \end{bmatrix} = t \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{b_1}$

$$\lambda = -9: \ A + 9I = \begin{bmatrix} -3+9 & 4 \\ 6 & -5+9 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 6 & 4 \end{bmatrix} \to \begin{array}{c} \begin{array}{cc} L & F \end{array} \\ \begin{bmatrix} 1 & 2/3 \\ 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_2 = t, \ x_1 = -\frac{2}{3}t \qquad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2/3\, t \\ t \end{bmatrix} = t \begin{bmatrix} -2/3 \\ 1 \end{bmatrix} = 3t \underbrace{\begin{bmatrix} -2 \\ 3 \end{bmatrix}}_{\vec{b}_2}$

$B = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix} \quad B^{-1} = \frac{1}{5} \begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix} \quad A_B = B^{-1} A B = \begin{bmatrix} 1 & 0 \\ 0 & -9 \end{bmatrix}$

c) $\vec{x}' = A\vec{x} \to \begin{pmatrix} \vec{x} = B\vec{y} \\ \vec{y} = B^{-1}\vec{x} \end{pmatrix} \to \vec{y}' = A_B \vec{y}$

$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -9 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ -9y_2 \end{bmatrix} \qquad \begin{array}{ll} y_1' = y_1 & y_1 = c_1 e^{t} \\ y_2' = -9y_2 & y_2 = c_2 e^{-9t} \end{array}$

$$\begin{bmatrix} -1 \\ 4 \end{bmatrix} = \vec{x}(0) = B \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \to \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 4 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} -3+8 \\ 1+4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} e^{t} \\ e^{-9t} \end{bmatrix} = e^{t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^{-9t} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \boxed{\begin{bmatrix} e^{t} - 2e^{-9t} \\ e^{t} + 3e^{-9t} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}$$

b) $\begin{bmatrix} 0 \\ 5 \end{bmatrix} = B \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \to \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

↑ oops, bob forgot to edit this to be $\langle -1, 4 \rangle$ as in IVP but no matter. makes sense anyway.

d) $x_2 = e^{t} + 3e^{-9t}$

$x_2' = e^{t} - 27e^{-9t} = 0 \xrightarrow{\times e^{9t}} e^{10t} - 27 = 0$

$10t = \ln 27 \qquad t = \frac{1}{10} \ln 27 = \frac{1}{10} \ln 3^3 = \boxed{\frac{3}{10} \ln 3 \approx 0.330}$

$x_2 = e^{\frac{1}{10} \ln 27} + 3e^{-\frac{9}{10} \ln 27}$

$= 27^{\frac{1}{10}} + 3(27)^{-9/10} = 27^{\frac{1}{10}} \left(1 + \frac{3}{(27)^{10/10}}\right)$

$\boxed{= \frac{30}{27^{9/10}} \approx 1.545} = 10 \cdot \frac{3}{3^{27/10}} = \frac{10 \cdot 3 \, 3^{3/10}}{3^{30/10}} = \frac{10}{9} 3^{3/10}$

→ local min at ≈ (0.33, 1.54) on graph