

CSE 321 Discrete Structures

Winter 2008
Lecture 1
Propositional Logic

About the course

- From the CSE catalog:
 - CSE 321 Discrete Structures (4)
Fundamentals of set theory, graph theory, enumeration, and algebraic structures, with applications in computing. Prerequisite: CSE 143; either MATH 126, MATH 129, or MATH 136.
- What I think the course is about:
 - Foundational structures for the practice of computer science and engineering

Why this material is important

- Language and formalism for expressing ideas in computing
- Fundamental tasks in computing
 - Translating imprecise specification into a working system
 - Getting the details right

Topic List

- Logic/boolean algebra: hardware design, testing, artificial intelligence, software engineering
- Mathematical reasoning/induction: algorithm design, programming languages
- Number theory/probability: cryptography, security, algorithm design, machine learning
- Relations/relational algebra: databases
- Graph theory: networking, social networks, optimization

Administration

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| <ul style="list-style-type: none">• Instructor<ul style="list-style-type: none">– Richard Anderson• Teaching Assistant<ul style="list-style-type: none">– Natalie Linnell• Quiz section<ul style="list-style-type: none">– Thursday, 12:30 – 1:20, or 1:30 – 2:20– CSE 305• Recorded Lectures<ul style="list-style-type: none">– Available on line• Text: Rosen, Discrete Mathematics<ul style="list-style-type: none">– 6th Edition preferred– 5th Edition okay | <ul style="list-style-type: none">• Homework<ul style="list-style-type: none">– Due Wednesdays (starting Jan 16)• Exams<ul style="list-style-type: none">– Midterms, Feb 8– Final, March 17, 2:30-4:20 pm• All course information posted on the web• Sign up for the course mailing list |
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Propositional Logic

Propositions

- A statement that has a truth value
- Which of the following are propositions?
 - The Washington State flag is red
 - It snowed in Whistler, BC on January 4, 2008.
 - Hillary Clinton won the democratic caucus in Iowa
 - Space aliens landed in Roswell, New Mexico
 - Ron Paul would be a great president
 - Turn your homework in on Wednesday
 - Why are we taking this class?
 - If n is an integer greater than two, then the equation $a^n + b^n = c^n$ has no solutions in non-zero integers a , b , and c .
 - Every even integer greater than two can be written as the sum of two primes
 - This statement is false
- Propositional variables: $p, q, r, s, \dots$
- Truth values: **T** for true, **F** for false

Compound Propositions

- Negation (not) $\neg p$
- Conjunction (and) $p \wedge q$
- Disjunction (or) $p \vee q$
- Exclusive or $p \oplus q$
- Implication $p \rightarrow q$
- Biconditional $p \leftrightarrow q$

Truth Tables

p	$\neg p$
T	F
F	T

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Understanding complex propositions

- Either Harry finds the locket and Ron breaks his wand or Fred will not open a joke shop

Understanding complex propositions with a truth table

p	r	f	$p \wedge r$	$\neg f$	$(p \wedge r) \oplus \neg f$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	F	F	T
F	T	F	F	T	T
F	F	T	F	F	T
F	F	F	F	T	T

Aside: Number of binary operators

- How many different binary operators are there on atomic propositions?

$$p \rightarrow q$$

p	q	$p \rightarrow q$

- Implication
 - p implies q
 - whenever p is true q must be true
 - if p then q
 - q if p
 - p is sufficient for q
 - p only if q

If pigs can whistle then horses
can fly

Converse, Contrapositive, Inverse

- Implication: $p \rightarrow q$
- Converse: $q \rightarrow p$
- Contrapositive: $\neg q \rightarrow \neg p$
- Inverse: $\neg p \rightarrow \neg q$
- Are these the same?

Biconditional $p \leftrightarrow q$

- p iff q
- p is equivalent to q
- p implies q and q implies p

p	q	$p \leftrightarrow q$

English and Logic

- You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old
 - q : you can ride the roller coaster
 - r : you are under 4 feet tall
 - s : you are older than 16

Logical equivalence

- Terminology: A compound proposition is a
 - Tautology if it is always true
 - Contradiction if it is always false
 - Contingency if it can be either true or false

$$p \vee \neg p$$

$$(p \oplus p) \vee p$$

$$p \oplus \neg p \oplus q \oplus \neg q$$

$$(p \rightarrow q) \wedge p$$

$$(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$$