CSE 321 Discrete Structures

Winter 2008
Lecture 1
Propositional Logic

About the course

- From the CSE catalog:
  - **CSE 321 Discrete Structures (4)**
    Fundamentals of set theory, graph theory, enumeration, and algebraic structures, with applications in computing. Prerequisite: CSE 143; either MATH 126, MATH 129, or MATH 136.
- What I think the course is about:
  - Foundational structures for the practice of computer science and engineering

Why this material is important

- Language and formalism for expressing ideas in computing
- Fundamental tasks in computing
  - Translating imprecise specification into a working system
  - Getting the details right

Topic List

- Logic/boolean algebra: hardware design, testing, artificial intelligence, software engineering
- Mathematical reasoning/induction: algorithm design, programming languages
- Number theory/probability: cryptography, security, algorithm design, machine learning
- Relations/relational algebra: databases
- Graph theory: networking, social networks, optimization

Administration

- Instructor
  - Richard Anderson
- Teaching Assistant
  - Natalie Linnell
- Quiz section
  - Thursday, 12:30 – 1:20, or 1:30 – 2:20
  - CSE 305
- Recorded Lectures
  - Available on line
- Text: Rosen, Discrete Mathematics
  - 6th Edition preferred
  - 5th Edition okay
- Homework
  - Due Wednesdays (starting Jan 16)
- Exams
  - Midterms, Feb 8
  - Final, March 17, 2:30-4:20 pm
- All course information posted on the web
- Sign up for the course mailing list

Propositional Logic

Propositions

- A statement that has a truth value
- Which of the following are propositions?
  - The Washington State flag is red
  - It snowed in Whistler, BC on January 4, 2008.
  - Hillary Clinton won the democratic caucus in Iowa
  - Space aliens landed in Roswell, New Mexico
  - Ron Paul would be a great president
  - Turn your homework in on Wednesday
  - Why are we taking this class?
  - If n is an integer greater than two, then the equation $a^n + b^n = c^n$ has no solutions in non-zero integers a, b, and c.
  - Every even integer greater than two can be written as the sum of two primes
  - This statement is false
- Propositional variables: $p, q, r, s, \ldots$
- Truth values: **T** for true, **F** for false

Compound Propositions

- Negation (not) $\neg p$
- Conjunction (and) $p \wedge q$
- Disjunction (or) $p \vee q$
- Exclusive or $p \oplus q$
- Implication $p \rightarrow q$
- Biconditional $p \leftrightarrow q$

Truth Tables

p	$\neg p$

p	q	$p \wedge q$

p	q	$p \vee q$

p	q	$p \oplus q$

Understanding complex propositions

- Either Harry finds the locket and Ron breaks his wand or Fred will not open a joke shop

Understanding complex propositions with a truth table

h	r	f	$h \wedge r$	$\neg f$	$(h \wedge r) \oplus \neg f$

Aside: Number of binary operators

- How many different binary operators are there on atomic propositions?

$p \to q$

p	q	$p \to q$

- Implication
  - p implies q
  - whenever p is true q must be true
  - if p then q
  - q if p
  - p is sufficient for q
  - p only if q

If pigs can whistle then horses can fly

Converse, Contrapositive, Inverse

- Implication: $p \to q$
- Converse: $q \to p$
- Contrapositive: $\neg q \to \neg p$
- Inverse: $\neg p \to \neg q$
- Are these the same?

Biconditional $p \leftrightarrow q$

- p iff q
- p is equivalent to q
- p implies q and q implies p

p	q	$p \leftrightarrow q$

English and Logic

- You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old
  - q: you can ride the roller coaster
  - r: you are under 4 feet tall
  - s: you are older than 16

Logical equivalence

- Terminology: A compound proposition is a
  - Tautology if it is always true
  - Contradiction if it is always false
  - Contingency if it can be either true or false

$p \vee \neg p$

$(p \oplus p) \vee p$

$p \oplus \neg p \oplus q \oplus \neg q$

$(p \to q) \wedge p$

$(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$