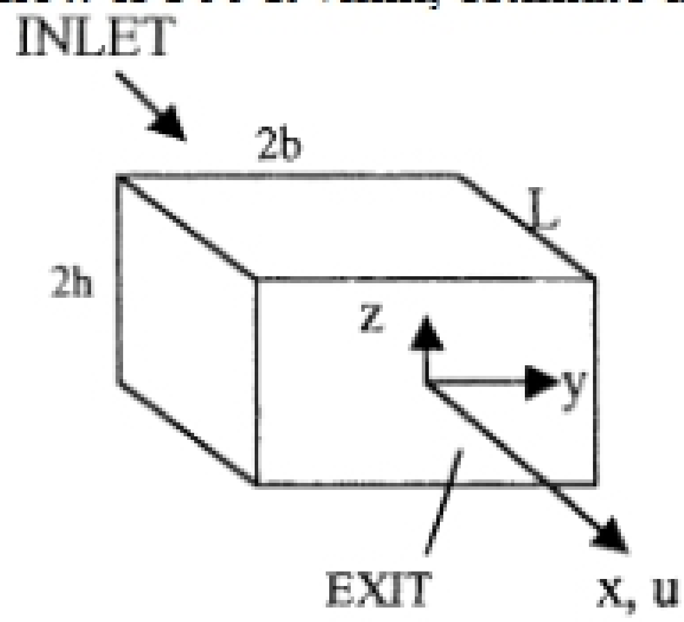
P3.18 An incompressible fluid flows steadily through the rectangular duct in the figure. The exit velocity profile is given by $u \approx u_{max}(1 - y^2/b^2)(1 - z^2/h^2)$. (a) Does this profile satisfy the correct boundary conditions for viscous fluid flow? (b) Find an analytical expression for the volume flow Q at the exit. (c) If the inlet flow is 300 ft^3/min, estimate u_{max} in m/s.



Solution: (a) The *fluid should not slip* at any of the duct surfaces, which are defined by $y = \pm b$ and $z = \pm h$. From our formula, we see **u ≡ 0 at all duct surfaces**, OK. *Ans.* (a)

(b) The exit volume flow Q is defined by the integral of u over the exit plane area:

$$Q = \iint u\, dA = \int_{-h}^{+h}\int_{-b}^{+b} u_{\max}\left(1-\frac{y^2}{b^2}\right)\left(1-\frac{z^2}{h^2}\right) dy\, dz = u_{\max}\int_{-b}^{+b}\left(1-\frac{y^2}{b^2}\right)dy\int_{-h}^{+h}\left(1-\frac{z^2}{h^2}\right)dz = u_{\max}\left(\frac{4b}{3}\right)\left(\frac{4h}{3}\right)$$

$$= \frac{\mathbf{16bhu_{max}}}{\mathbf{9}} \quad \textit{Ans. (b)}$$

(c) Given $Q = 300$ ft^3/min $= 0.1416$ m^3/s and $b = h = 10$ cm, the maximum exit velocity is

$$Q = 0.1416\ \frac{\text{m}^3}{\text{s}} = \frac{16}{9}(0.1\text{ m})(0.1\text{ m})u_{\max}, \quad \textit{solve for } \mathbf{u_{max} = 7.96\ m/s} \quad \textit{Ans. (c)}$$

P3.33 In some wind tunnels the test section is perforated to suck out fluid and provide a thin viscous boundary layer. The test section wall in Fig. P3.33 contains 1200 holes of 5-mm diameter each per square meter of wall area. The suction velocity through each hole is $V_r = 8$ m/s, and the test-section entrance velocity is $V_1 = 35$ m/s. Assuming incompressible steady flow of air at 20°C, compute (a) V_o, (b) V_2, and (c) V_f, in m/s.

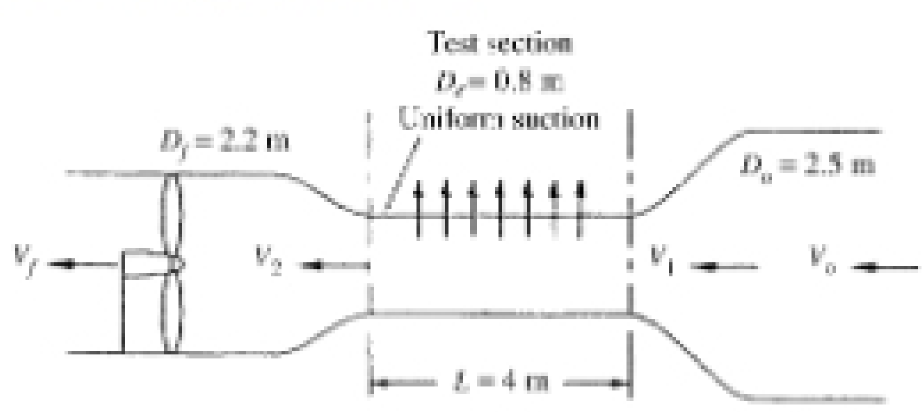


Fig. P3.33

$$\text{Area of test section} = 2\pi rL = 2\pi(0.4m)(4m) = 10.053m^2$$

The number of holes on the test section is $1m^2 : 1200 = 10.0531 : N,\ N = 12064$

$$Q_{suction} = NQ_{hole} = NAV_r = (12064)(\pi/4)(0.005\ \text{m})^2(8\ \text{m/s}) \approx 1.895\ \text{m}^3/\text{s}$$

$$\text{(a)}\quad \text{Find } V_o:\quad Q_o = Q_1 \quad \text{or} \quad V_o\frac{\pi}{4}(2.5)^2 = (35)\frac{\pi}{4}(0.8)^2,$$

$$\text{solve for}\quad V_o \approx \mathbf{3.58}\ \frac{\textbf{m}}{\textbf{s}} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{(b)}\quad Q_2 = Q_1 - Q_{suction} = (35)\frac{\pi}{4}(0.8)^2 - 1.895 = V_2\frac{\pi}{4}(0.8)^2,$$

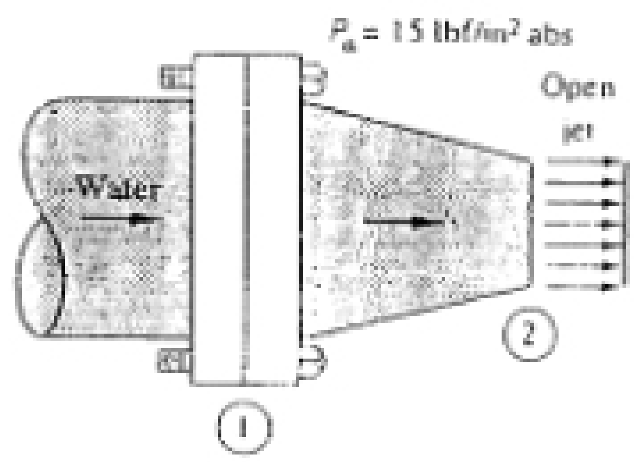
$$\text{or:}\quad V_2 \approx \mathbf{31.2}\ \frac{\textbf{m}}{\textbf{s}} \quad \textit{Ans.}\ \text{(b)}$$

$$\text{(c)}\quad \text{Find } V_f:\quad Q_f = Q_2 \quad \text{or} \quad V_f\frac{\pi}{4}(2.2)^2 = (31.2)\frac{\pi}{4}(0.8)^2,$$

$$\text{solve for}\quad V_f \approx \mathbf{4.13}\ \frac{\textbf{m}}{\textbf{s}} \quad \textit{Ans.}\ \text{(c)}$$

P3.49 The horizontal nozzle in Fig. P3.49 has D1 = 12 in, D2 = 6 in, with p1 = 38 psia and V2 = 56 ft/s. For water at 20°C, find the force provided by the flange bolts to hold the nozzle fixed.

Solution: For an open jet, p2 = pa = 15 psia. Subtract pa everywhere so the only nonzero pressure is p1 = 38 − 15 = 23 psig.



The mass balance yields the inlet velocity:

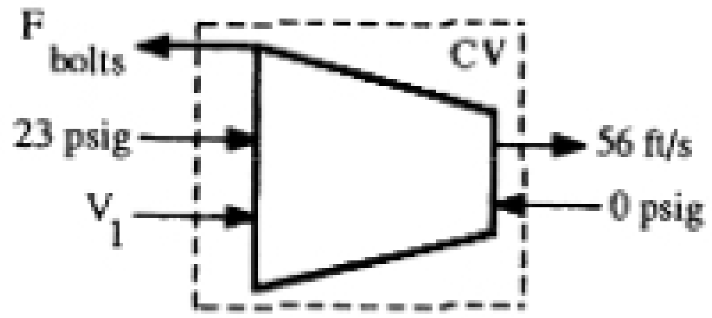
$$V_1A_1 = V_2A_2,\ V_1\frac{\pi}{4}(12)^2 = (56)\frac{\pi}{4}(6)^2,\quad V_1 = 14\ \frac{\text{ft}}{\text{s}}$$

The density of water is 1.94 slugs per cubic foot. Then the horizontal force balance is

$$\sum F_X = \dot{m}_2u_2 - \dot{m}_1u_1$$

$$\sum F_x = -F_{bolts} + (23\ \text{psig})\frac{\pi}{4}(12\ \text{in})^2 = \dot{m}_2u_2 - \dot{m}_1u_1 = \dot{m}(V_2 - V_1)$$

Compute $F_{bolts} = 2601 - (1.94)\frac{\pi}{4}(1\text{ ft})^2\left(14\ \frac{\text{ft}}{\text{s}}\right)\left(56 - 14\ \frac{\text{ft}}{\text{s}}\right) \approx \mathbf{1700\ lbf}$ *Ans.*



P3.61 A 20°C water jet strikes a vane on a tank with frictionless wheels, as shown. The jet turns and falls into the tank without spilling. If $\theta = 30°$, estimate the horizontal force F needed to hold the tank stationary.

Solution: The CV surrounds the tank and wheels and cuts through the jet, as shown. We should *assume that the splashing into the tank does not increase the x-momentum of the water in the tank*. Then we can write the CV horizontal force relation:

$$\sum F_x = -F = \frac{d}{dt}\left(\int u\rho\, d\upsilon\right)_{tank} - \dot{m}_{in}u_{in} = 0 - \dot{m}V_{jet} \text{ independent of } \theta$$

$$\text{Thus}\quad F = \rho A_j V_j^2 = \left(1.94\ \frac{\text{slug}}{\text{ft}^3}\right)\frac{\pi}{4}\left(\frac{2}{12}\ \text{ft}\right)^2\left(50\ \frac{\text{ft}}{\text{s}}\right)^2 \approx \mathbf{106\ lbf}\quad \textit{Ans.}$$

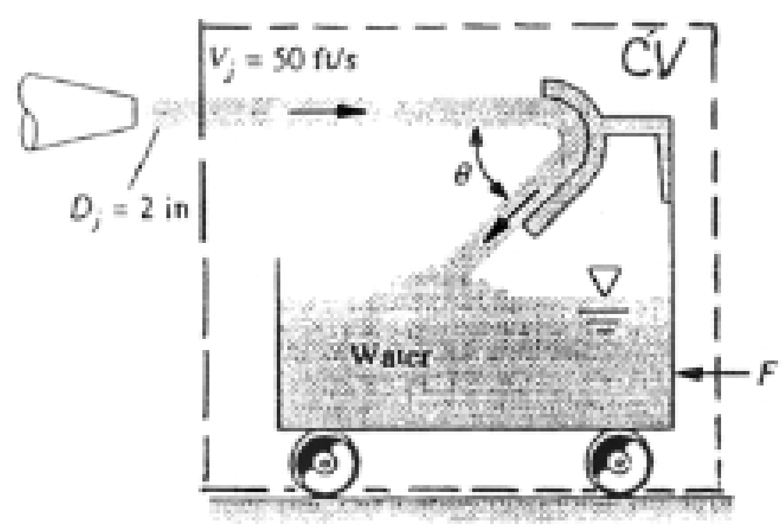


Fig. P3.61