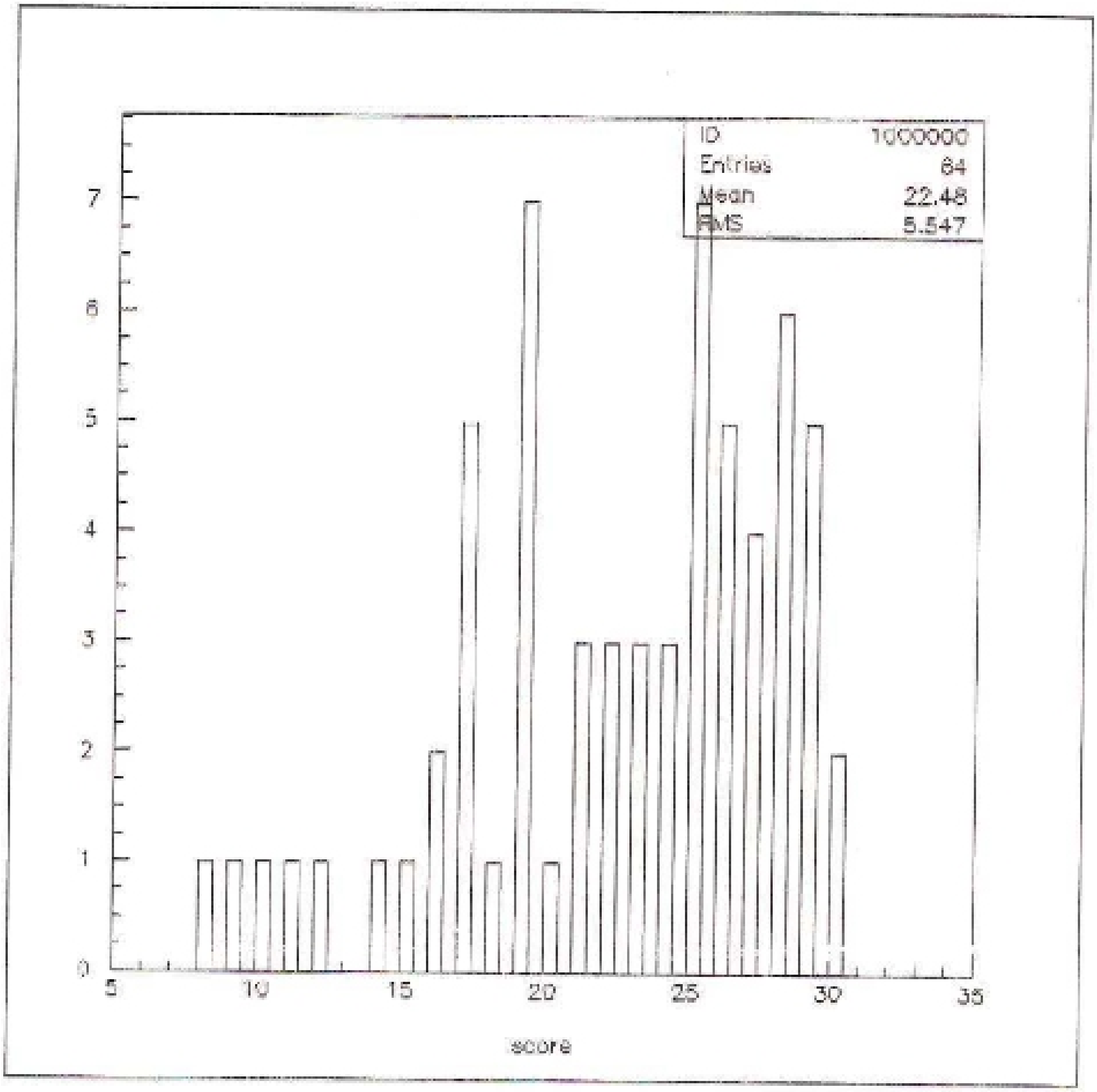


Second midterm score distribution

History

Johann Bernoulli posed the problem of the brachystochrone to the readers of *Acta Eruditorum* in June, 1696. He published his solution in the journal in May of the following year, and noted that the solution is the same curve as Huygens's tautochrone curve. After deriving the differential equation for the curve by the method given above, he went on to show that it does yield a cycloid.[1][2] But his proof is marred by the fact that he uses a single constant instead of the three constants, v_m, $2g$ and D, above. Five mathematicians responded with solutions: Isaac Newton, Jakob Bernoulli (Johann's brother), Gottfried Leibniz, Ehrenfried Walther von Tschirnhaus and Guillaume de l'Hôpital. Four of the solutions (excluding l'Hôpital's) were published in the same edition of the journal as Johann Bernoulli's. In his paper Jakob Bernoulli gave a proof of the condition for least time similar to that above before showing that its solution is a cycloid.[1]

In an attempt to outdo his brother, Jakob Bernoulli created a harder version of the brachistochrone problem. In solving it, he developed new methods that were refined by Leonhard Euler into what the latter called (in 1766) the *calculus of variations*. Joseph-Louis de Lagrange did further work that resulted in modern infinitesimal calculus.

Galileo tried to solve a similar problem for the path of the fastest descent from a point to a wall in his *Two New Sciences* in 1638. He draws the conclusion (Third Day, Theorem 22, Prop. 36) that the arc of a circle is faster than any number of its chords,[3]

> "From the preceding it is possible to infer that the quickest path of all [lationem omnium velocissimam], from one point to another, is not the shortest path, namely, a straight line, but the arc of a circle.
> ...
> Consequently the nearer the inscribed polygon approaches a circle the shorter is the time required for descent from A to C. What has been proven for the quadrant holds true also for smaller arcs; the reasoning is the same."

We are warned earlier in the Discourses (just after Theorem 6) of possible fallacies and the need for a "higher science." In this dialogue Galileo reviews his own work. The actual solution to Galileo's problem is half a cycloid. Galileo studied the cycloid and gave it its name, but the connection between it and his problem had to wait for advances in mathematics.

See also

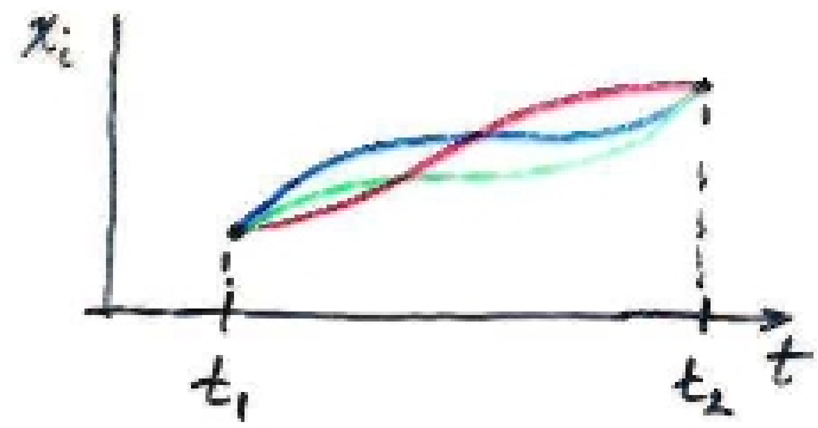
- Calculus of variations

Hamilton's Principle

Of all the possible paths along which a dynamical system may move from one point to another within a specified time interval (consistent with any constraints), the actual path followed is that which minimizes the time integral of the difference between the kinetic and potential energies.

$$L(x_i, \dot{x}_i; t) = T - U$$

$$\delta \int_{t_1}^{t_2} L \, dt = 0$$



"Extremum among all paths with the fixed endpoints"

That is to say:

Consider any family of paths (labeled by α) given by any functions $\eta_i(t)$ subject only to $\eta_i(t_1) = 0 = \eta_i(t_2)$:

$$x_i(t, \alpha) = x_i(t, 0) + \alpha \, \eta_i(t)$$

For all such families we must have $\frac{d}{d\alpha} \int_{t_1}^{t_2} L(\alpha) \, dt = 0$

Note: L has a unique value for each t, but it is not a simple function of t. It is a <u>functional</u> (depends on functions of t).

$$L = L(x_i(t), \dot{x}_i(t); t)$$

The "Lagrangian"