Steps for solving problems in Lagrangian mechanics

1. Write T and $\mathcal{U}$ in any convenient <u>inertial</u> frame.

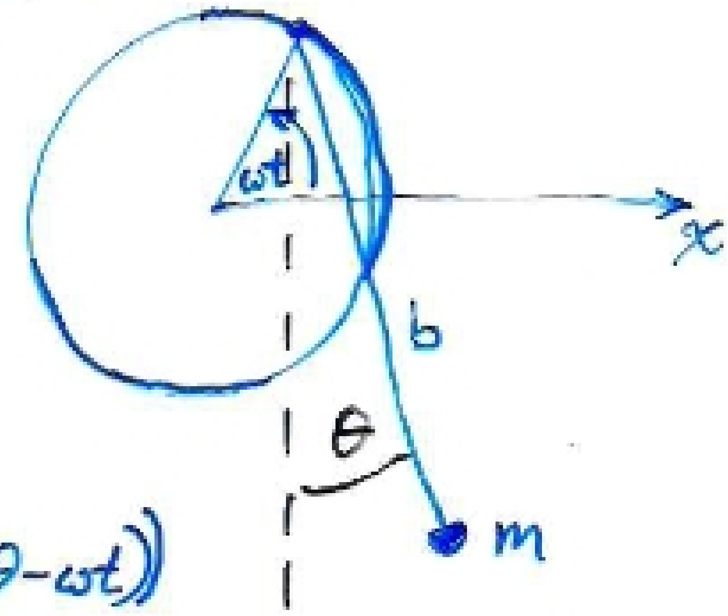
2. Choose suitable generalized coordinates q_i to describe the possible configurations

3. Write $L \equiv T - \mathcal{U}$ in terms of $q_i, \dot{q}_i, t$.

4. Work out the resulting Euler-Lagrange equations of motion.

(5) (Solve the equations, or answer the questions, as required.)

Example 7.5
See p.242



Massless rim $\Rightarrow$
all energy due to the "bob" m.

$$T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2)$$
$$= \frac{1}{2} m (a^2\omega^2 + b^2\dot{\theta}^2 + 2b\dot{\theta}a\omega\sin(\theta-\omega t))$$

$$U = mgy = mg(a\sin\omega t - b\cos\theta)$$

$$L = T - U$$

$$\frac{\partial L}{\partial \dot{\theta}} = mb^2\dot{\theta} + mba\omega\sin(\theta-\omega t)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = mb^2\ddot{\theta} + mba\omega(\dot{\theta}-\omega)\cos(\theta-\omega t)$$

$$\frac{\partial L}{\partial \theta} = mb\dot{\theta}a\omega\cos(\theta-\omega t) - mgb\sin\theta$$

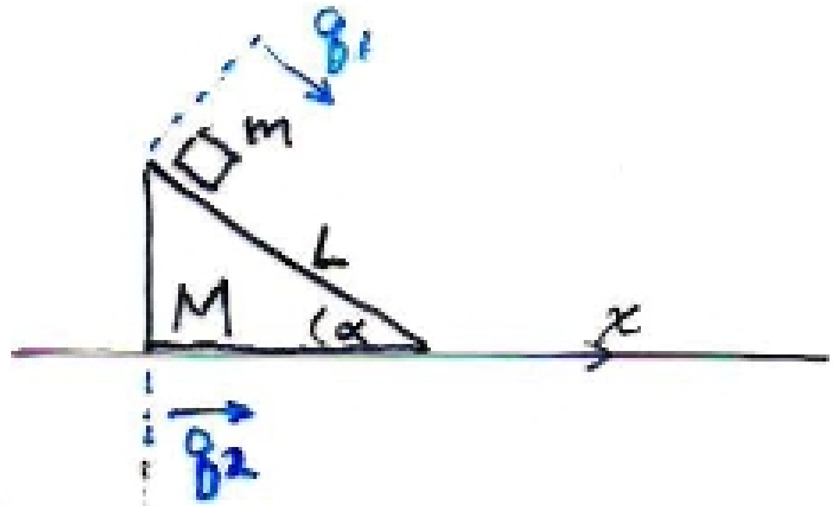
Euler-Lagrange $\quad \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0$

$\Downarrow$

$$\ddot{\theta} = \frac{a\omega^2}{b}\cos(\theta-\omega t) - \frac{g}{b}\sin\theta$$

Simple pendulum for $\omega = 0$.
Otherwise
$$\ddot{\theta} + \frac{g}{b}\sin\theta = \frac{a\omega^2}{b}\cos(\theta-\omega t)$$

Block m slides on wedge of mass M (also sliding). How long does it take to slide distance L ?



$$T_W = \frac{1}{2} M \dot{x}_W^2 \qquad U_W = \text{const}$$

$$T_B = \frac{1}{2} m (\dot{x}_B^2 + \dot{y}_B^2) \qquad U_B = +mgy + \text{const}$$

$$x_W = q_2 \qquad x_B = q_1 \cos\alpha + q_2 \qquad y_B = y_0 - q_1 \sin\alpha$$

$$T = T_W + T_B = \frac{1}{2} M \dot{q}_2^2 + \frac{1}{2} m (\dot{q}_1^2 + \dot{q}_2^2 + 2\dot{q}_1 \dot{q}_2 \cos\alpha)$$

$$U = -mg q_1 \sin\alpha + \text{const}$$

$$L = \frac{1}{2}(M+m)\dot{q}_2^2 + \frac{1}{2} m (\dot{q}_1^2 + 2\dot{q}_1 \dot{q}_2 \cos\alpha) + mg q_1 \sin\alpha$$

$$\frac{\partial L}{\partial q_2} = 0 \Rightarrow \frac{\partial L}{\partial \dot{q}_2} = \text{const} \Rightarrow M\dot{q}_2 + m(\dot{q}_2 + \dot{q}_1 \cos\alpha) = \text{const}$$

$$\Rightarrow M\ddot{q}_2 + m\ddot{q}_2 + m\ddot{q}_1 \cos\alpha = 0$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_1} - \frac{\partial L}{\partial q_1} = 0 \Rightarrow m(\ddot{q}_1 + \ddot{q}_2 \cos\alpha) = mg \sin\alpha$$

$$\ddot{q}_1 = \frac{g \sin\alpha}{1 - \frac{m\cos^2\alpha}{M+m}}$$

constant acceleration a

$$L = \frac{1}{2} a t^2 \Rightarrow t = \sqrt{\frac{2L}{a}}$$