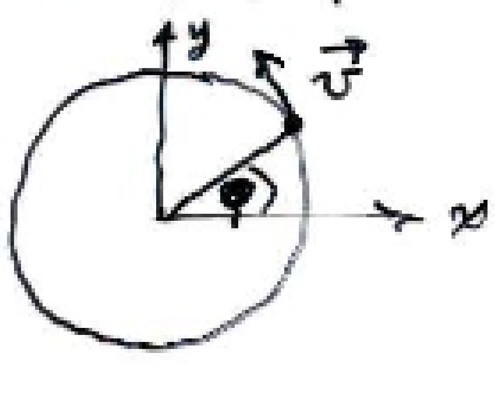
Angular velocity and acceleration

Simple circular motion about the origin in the xy plane.

$\theta \equiv 0, \quad r = r_0$ (constant), $|\vec{v}| = v$ (constant)

Rotation period $\tau = \dfrac{2\pi r_0}{v}$



$$\omega \equiv \frac{d\varphi}{dt} = \frac{2\pi}{\tau} = \frac{v}{r_0}$$

$$\Leftrightarrow \boxed{v = r_0 \omega}$$

$$\vec{a} \equiv \ddot{\vec{r}} = \text{(see general expression)}$$

$$= -r_0 \dot{\varphi}^2 \vec{e}_r$$

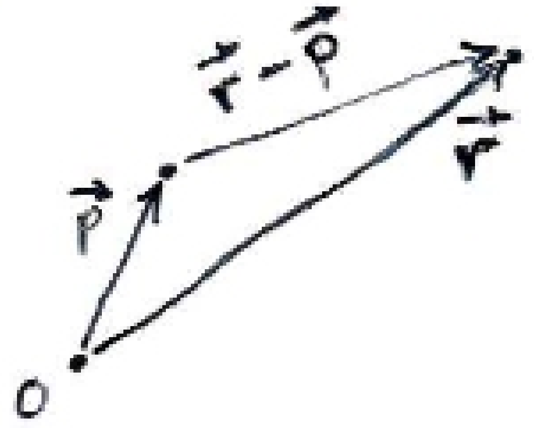
So $\quad a = \omega^2 r_0 = \dfrac{v^2}{r_0}, \qquad \vec{a} = -\dfrac{v^2}{r_0} \vec{e}_r$

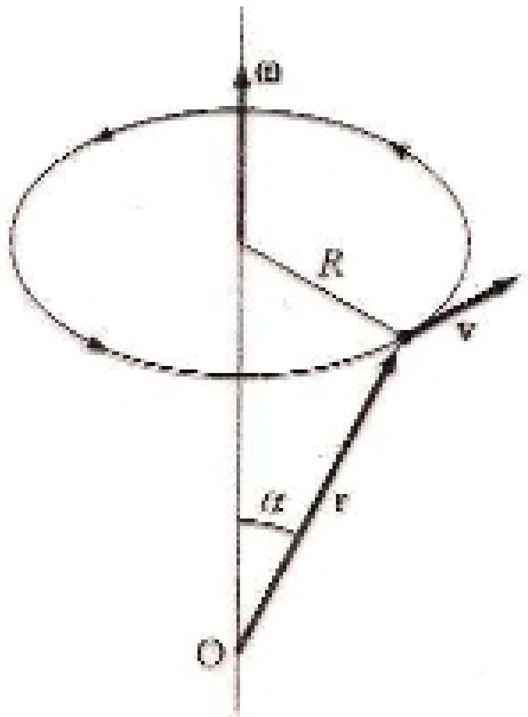
In general $\vec{v} = \vec{\omega} \times \vec{r}$ and $\vec{\omega} = \dfrac{\vec{r} \times \vec{v}}{r^2}$

Angular velocity depends on the reference point.

About point $\vec{p}$, the angular velocity is

$$\vec{\omega} = \frac{(\vec{r} - \vec{p}) \times \vec{v}}{|\vec{r} - \vec{p}|^2}$$





$$\vec{\omega} = \frac{\vec{r} \times \vec{v}}{r^2}$$

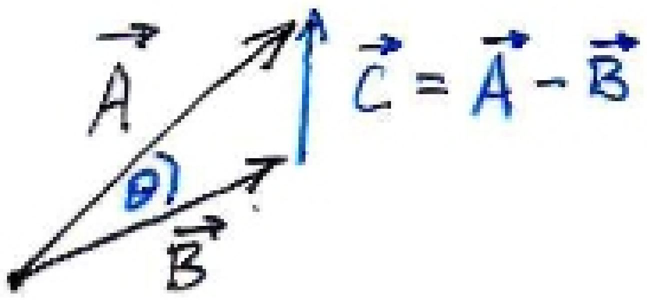
FIGURE 1-18 A particle moving ccw about an axis according to the right-hand rule has an angular velocity $\boldsymbol{\omega} \equiv \mathbf{v} \times \mathbf{r}$ about that axis.

Having defined a direction and a magnitude for the angular velocity, we note that if we write

$$\boxed{\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}} \tag{1.105}$$

1-17. Obtain the cosine law of plane trigonometry by interpreting the product $(\mathbf{A} - \mathbf{B}) \cdot (\mathbf{A} - \mathbf{B})$ and the expansion of the product.

$$\vec{A} \cdot \vec{B} = AB \cos\theta$$



$$\vec{C} \cdot \vec{C} = (\vec{A} - \vec{B}) \cdot (\vec{A} - \vec{B}) = \vec{A} \cdot \vec{A} + \vec{B} \cdot \vec{B} - 2\vec{A} \cdot \vec{B}$$

$$C^2 = A^2 + B^2 - 2AB \cos\theta$$