Chapter 1 Review

$$\vec{A}\cdot\vec{B} = AB\cos\theta = A_1B_1 + A_2B_2 + A_3B_3$$

$|\vec{A}\times\vec{B}| = AB\sin\theta$, $\vec{A}\times\vec{B}$ direction given by right hand rule

$$\vec{A}\times\vec{B} = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix} \qquad A_i = \vec{A}\cdot\vec{e}_i \qquad B_i = \vec{B}\cdot\vec{e}_i$$

In Cartesian coordinates x, y, z:

Grad: $$\vec{\nabla} f = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}$$

Div: $$\vec{\nabla}\cdot\vec{A} = \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}$$

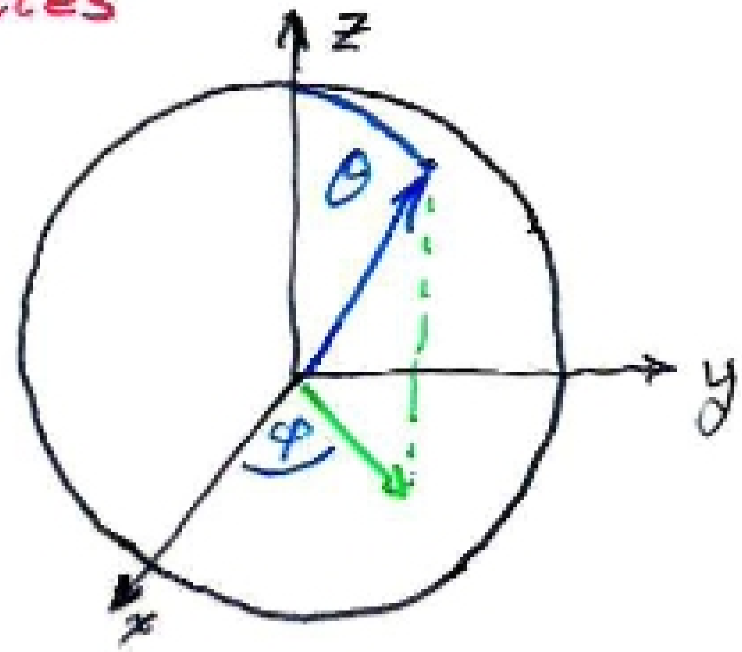
Curl: $$\vec{\nabla}\times\vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$$

Laplacian $$\nabla^2 f \equiv \vec{\nabla}\cdot\vec{\nabla} f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Gauss' Law: $$\int_V \vec{\nabla}\cdot\vec{A}\, dv = \int_S \vec{A}\cdot\vec{n}\, da$$

Stokes' Theorem: $$\int_S \vec{\nabla}\times\vec{A}\, da = \int_C \vec{A}\cdot\vec{T}\, ds$$

Spherical Polar Coordinates



$r = \sqrt{x^2+y^2+z^2}$

$\theta = \cos^{-1}\left(z/\sqrt{x^2+y^2+z^2}\right)$

$\varphi = \tan^{-1}(y/x)$

$x = r\sin\theta\cos\varphi$

$y = r\sin\theta\sin\varphi$

$z = r\cos\theta$

Note: Derivatives of $\vec{i}$, $\vec{j}$, or $\vec{k}$ are 0.

But derivatives of $\vec{e}_r$, $\vec{e}_\theta$, $\vec{e}_\varphi$ are not 0 (in general)

Position vector is $r\vec{e}_r$

$\frac{d}{dt}\vec{e}_r \neq 0$, so velocity (and acceleration) have $\vec{e}_\theta$ and $\vec{e}_\varphi$ components (in general).

$\vec{\nabla}f$, $\vec{\nabla}\cdot\vec{A}$, and $\vec{\nabla}\times\vec{A}$ are more complicated than in Cartesian x,y,z coordinates. (See appendix F)

Chapter 2 Review

$\vec{F} = m\vec{a}$

Solve for $\vec{r}(t)$, position as function of time, given a starting position $\vec{r}(0)$ and initial velocity $\vec{v}(0)$, using a known force $\vec{F}(\vec{r})$.

Examples

Sliding on an inclined plane
Projectile motion
Weight with ropes and pulleys.

Conservative force: $\vec{F} = -\vec{\nabla} U \iff \vec{\nabla} \times \vec{F} = 0$

Examples: gravity $\vec{F} = -mg\vec{k}$ $U = mgz$
spring $\vec{F} = -k(x - x_0)$ $U = \frac{1}{2}k(x-x_0)^2$

Non-conservative force $\vec{\nabla} \times \vec{F} \neq 0$, no potential energy.

Examples: Friction
Air resistance

Work $W = \int_1^2 \vec{F} \cdot \vec{dr}$

Path independence $\iff$ "work function" $\iff$ potential energy

$T + U$ = constant if motion is governed by a conservative force with potential energy U.

T = kinetic energy = $\frac{1}{2}mv^2$