Overview of $\vec{F}=m\vec{a}$ example problems

Section 2.4

* Blocks on inclined planes

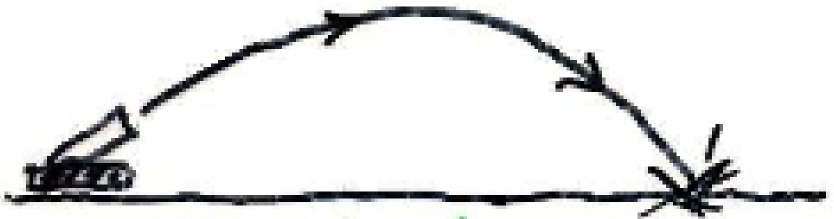
Gravitational force components parallel ($mg\sin\theta$) and perpendicular ($mg\cos\theta$) to the plane

Normal force $\vec{N}$ ($|\vec{N}| = mg\cos\theta$)

Coefficient of static friction μ_s

Coefficient of kinetic friction μ_k

* Projectile motion



Uniform acceleration $-g$ vertically due to gravity.

Zero horizontal acceleration from gravity.

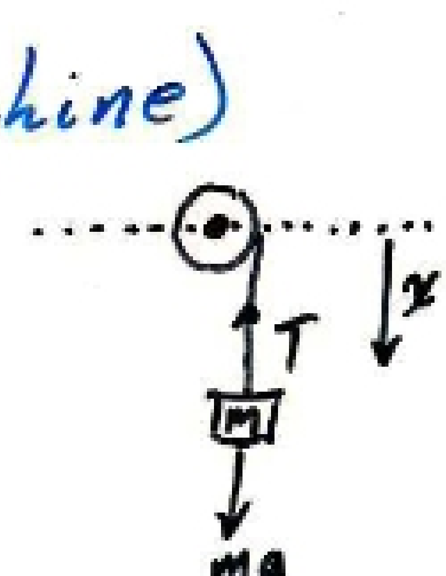
Resistive (drag) forces, usually

$$\vec{F} = -mk v^n \cdot \vec{v}/|\vec{v}|$$

Common example ($n=1$):

$$\vec{F} = -mk\vec{v}$$

* Hanging weights (Atwood's machine)



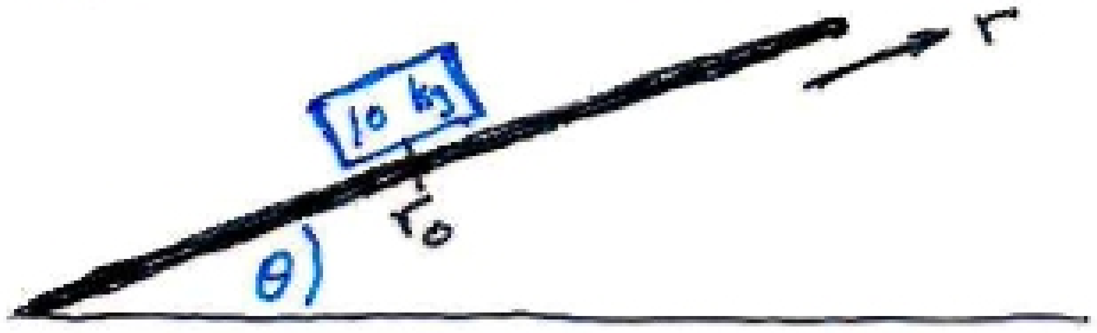
Frictionless (non-moving) pulley.

For any mass

$$m\ddot{x} = mg - T$$

(T = rope tension)

Sliding on an inclined plane



A 10-kg block moves on an inclined plane. The plane makes an angle θ with the horizontal.

The coefficient of static friction is $\mu_s = 0.5$

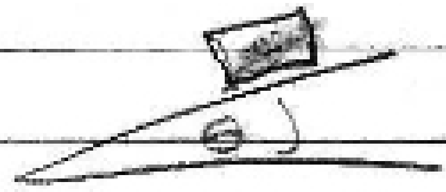
The coefficient of kinetic friction is $\mu_k = 0.3$

At $t=0$, the block is at $r = r_0$ and it is moving uphill with speed $v_0 = 5$ m/s.

Describe its motion $r(t)$ quantitatively for $t > 0$:

(A) if $\theta = 20°$
(B) if $\theta = 30°$

Inclined plane notes



Going uphill until $v = 0$.

$$F = -mg\sin\theta - mg\,\mu_k\cos\theta$$

$$a_0 = -g(\sin\theta + \mu_k\cos\theta) \equiv a_0 \quad \underline{\text{constant}}$$

$$v(t) = v_0 + \int_0^t a_0\,dt' = v_0 + a_0 t$$

$$r(t) = r_0 + \int_0^t v(t')\,dt' = r_0 + v_0 t + \tfrac{1}{2}a_0 t^2$$

Evaluate a_0 & solve for t_1: $v_0 + a_0 t_1 = 0$

$$t_1 = -v_0/a_0$$

$$r_1 \equiv r(t_1) = r_0 + v_0 t_1 + \tfrac{1}{2}a_0 t_1^2$$

Stopped at time t_1

$$F_{down} = -mg\sin\theta$$

$$F_{up} = +mg\,\mu_s\cos\theta$$

$\theta = 20°$ $\quad |F_{up}| = .47\,mg$

$|F_{down}| = .34\,mg$

Does not move for $t > t_1$

$\theta = 30°$ $\quad |F_{up}| = .43\,mg$

$|F_{down}| = .50\,mg$

Slides

$$F = -mg\sin\theta + mg\,\mu_k\cos\theta$$

$$a = -g(\sin\theta - \mu_k\cos\theta) \equiv a_0$$

$$r(t) = r_1 + 0(t-t_1) + \tfrac{1}{2}a_0(t-t_1)^2$$

Evaluate: $r_2 = r_0 \Rightarrow \tfrac{1}{2}a_0(t_2-t_1)^2 = r_0 - r_1$ $\quad \boxed{t_2 = t_1 + \sqrt{\dfrac{2(r_0-r_1)}{a_0}}}$