Math 364: Principles of Optimization, Lecture 5

Haijun Li

lih@math.wsu.edu
Department of Mathematics
Washington State University

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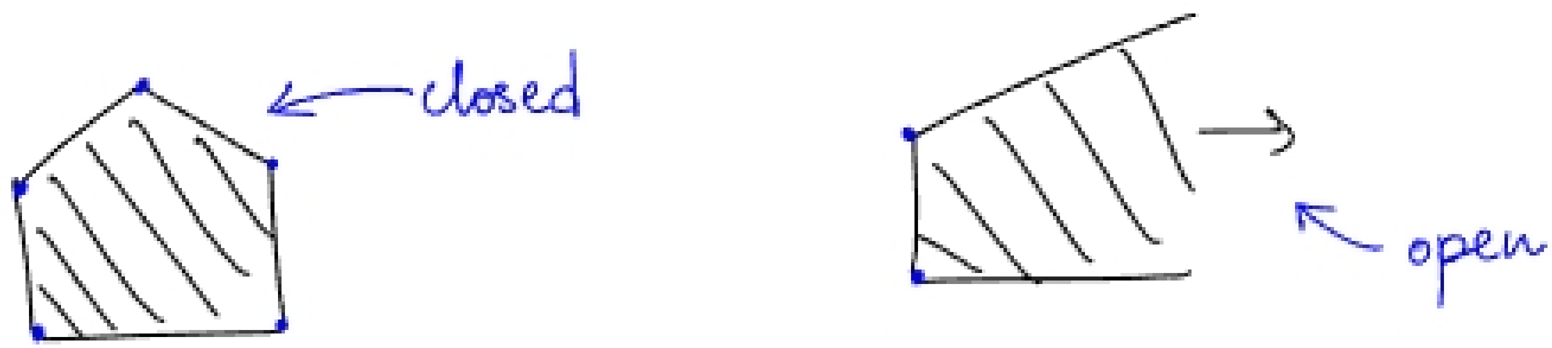
Convex Feasible Regions

A subset $\mathbb{S} \subseteq \mathbb{R}^n$ is *convex* if

$$\bar{\mathbf{v}}_1, \bar{\mathbf{v}}_2 \in \mathbb{S}, \text{ and } 0 \leq \alpha \leq 1 \;\Rightarrow\; \alpha\bar{\mathbf{v}}_1 + (1-\alpha)\bar{\mathbf{v}}_2 \in \mathbb{S}$$

LP feasible regions are convex, and are closed or open polyhedra, e.g., ← convex polygons in 2D



Corner points of the feasible region of an LP are called as **extreme points** or **vertices**.

Cases of Linear Programming Problems

Case 1: The LP has a **unique optimal solution**.
For example, the Farmer Jones LP.

Result When there is a unique optimal solution for an LP, it occurs at an extreme point.

Case 2: Some LPs have infinitely many optimal solutions, or **alternative optimal solutions**.

Case 3: Some LPs have no feasible solutions, and hence no optimal solutions. These **LPs are infeasible**. The feasible region is empty.