MA 15200 Lesson 28 Section 4.2

Remember the following information about inverse functions.

1. In order for a function to have an inverse, it must be one-to-one and pass a horizontal line test.
2. The inverse function can be found by interchanging *x* and *y* in the function's equation and solving for *y*.
3. If $f(a) = b$, then $f^{-1}(b) = a$. The domain of *f* is the range of f^{-1} and the range of *f* is the domain of f^{-1}.
4. The compositions $f(f^{-1}(x))$ and $f^{-1}(f(x))$ both equal *x*.
5. The graph of f^{-1} is the reflection of the graph of *f* about the line $y = x$.

Because an exponential function is 1-1 and passes the horizontal line test, it has an inverse. This inverse is called a logarithmic function.

I Logarithmic Functions

According to point 2 above, we interchange the *x* and *y* and solve for *y* to find the equation of an inverse function.

$f(x) = b^x$ exponential function

$x = b^y$ inverse function How do we solve for *y*? There is no way to do this. Therefore a new notation needs to be used to represent an inverse of an exponential function, the logarithmic function.

Definition of Logarithmic Function
For $x > 0$ and $b > 0$ $(b \neq 1)$
$y = \log_b x$ is equivalent to $x = b^y$
The function $f(x) = \log_b x$ is the **logarithmic function with base *b*.**

The equation $y = \log_b x$ is called the logarithmic form and the equation $x = b^y$ is called the exponential form. The value of *y* in either form is called a **logarithm.** Note: The logarithm is an exponent.

Exponential Form

$x = b^y$

argument base exponent

Logarithmic Form

$y = \log_b x$

exponent base argument

Ex 1: Convert each exponential form to logarithmic form and each logarithmic form to exponential form.

$a)\quad 3^4 = 81$

$b)\quad \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

$c)\quad 25^{\frac{1}{2}} = 5$

$d)\quad 8^{-2} = \frac{1}{64}$

$e)\quad \log_2 32 = 5$

$f)\quad \log_{\frac{1}{2}}\left(\frac{1}{8}\right) = 3$

$g)\quad \log_5 \sqrt{5} = \frac{1}{2}$

$h)\quad m^{rp} = x + 4$

$i)\quad (2a)^{y+7} = p^2$

$j)\quad \log_q(2mn) = 12$

$k)\quad \log_{x+3} 200 = rs$

II Finding logarithms

Remember: A logarithm is an exponent.

Ex 2: Find each logarithm.

$a)\quad \log_{10} 100,000$

$b)\quad \log_3 27$

$c)\quad \log_{20} 1$

$d)\quad \log_{15} 15$

$e)\quad \log_{12} \frac{1}{144}$

$f)\quad \log_4 64$

$g)\quad \log_{\frac{1}{2}} 32$

$h)\quad \log_3 81$

III Basic Logarithmic Properties

1. $\log_b b = 1$ Since the first power of any base equals that base, this is reasonable.
2. $\log_b 1 = 0$ Since any base to the zero power is 1, this is reasonable.

The exponential function $f(x) = b^x$ or $y = b^x$ and the logarithmic function $f^{-1}(x) = \log_b x$ or $y = \log_b x$ are inverses.
$f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$

This leads to 2 more basic logarithmic properties.

3. $\log_b b^x = x$ This is a composition function where $f(x) = b^x$ and $f^{-1}(x) = \log_b x$. $f^{-1}(f(x)) = f^{-1}(b^x) = \log_b b^x = x$ (the exponent)
4. $b^{\log_b x} = x$ This is a composition function where $f(x) = b^x$ and $f^{-1}(x) = \log_b x$. $f(f^{-1}(x)) = f(\log_b x) = b^{\log_b x} = x$ (the number or argument)

Ex 3: Simplify using the basic properties of logarithms.

$a)\quad \log_4 1 =$

$b)\quad \log_3 3 =$

$c)\quad 12^{\log_{12} 4} =$

$d)\quad \log_{10} 10^5 =$