6.3 Cylindrical Shell method

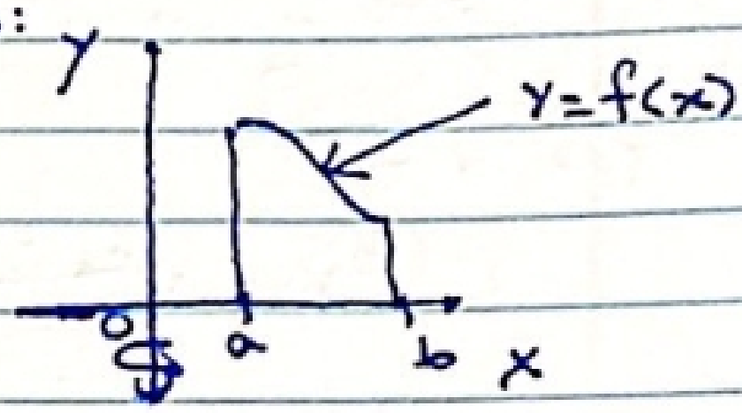
→ Last section we learned method

- Disk $V = \int_a^b \pi r^2 dx$
- Washer $V = \int_a^b (\pi R^2 - \pi r^2) dx$

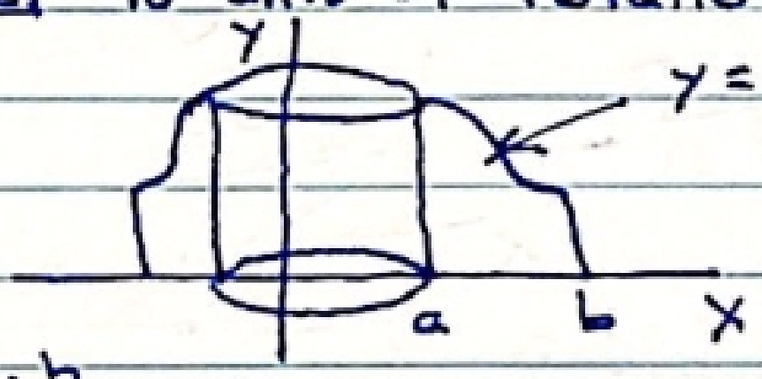
→ Slicing <u>perpendicular</u> to axis of rotation

We will learn now SHELL METHOD:

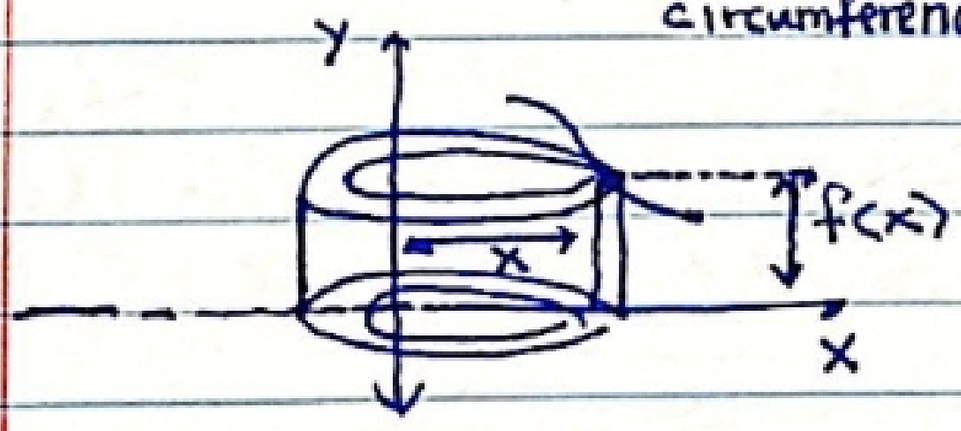
$$V = \int_a^b 2\pi r h \, dx$$

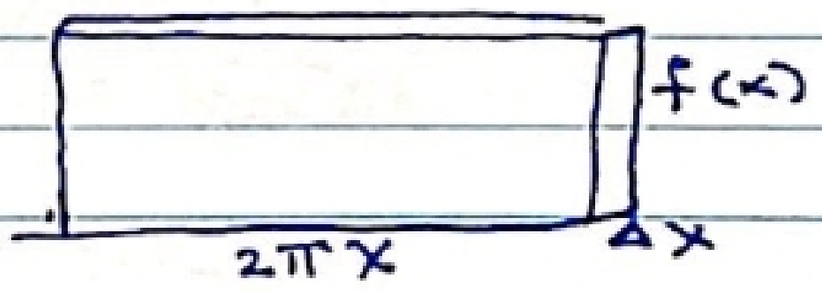


peeling <u>parallel</u> to axis of rotation



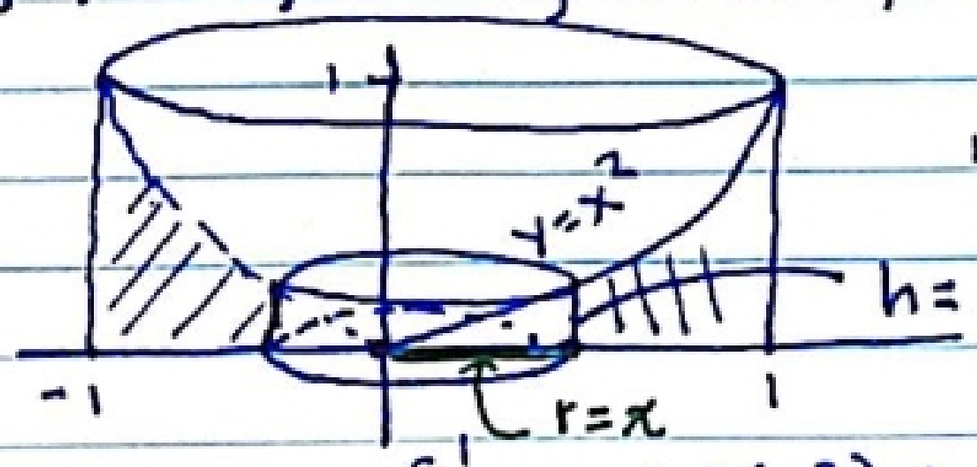
$$V = \int_a^b \underbrace{(2\pi x)}_{\text{circumference}} \underbrace{(f(x))}_{\text{height}} \underbrace{dx}_{\text{thickness}}$$





<u>Example</u>: Find volume. Sketch the region and a typical Shell.

1) $y = x^2$, $y = 0$, $x = 1$, about y-axis



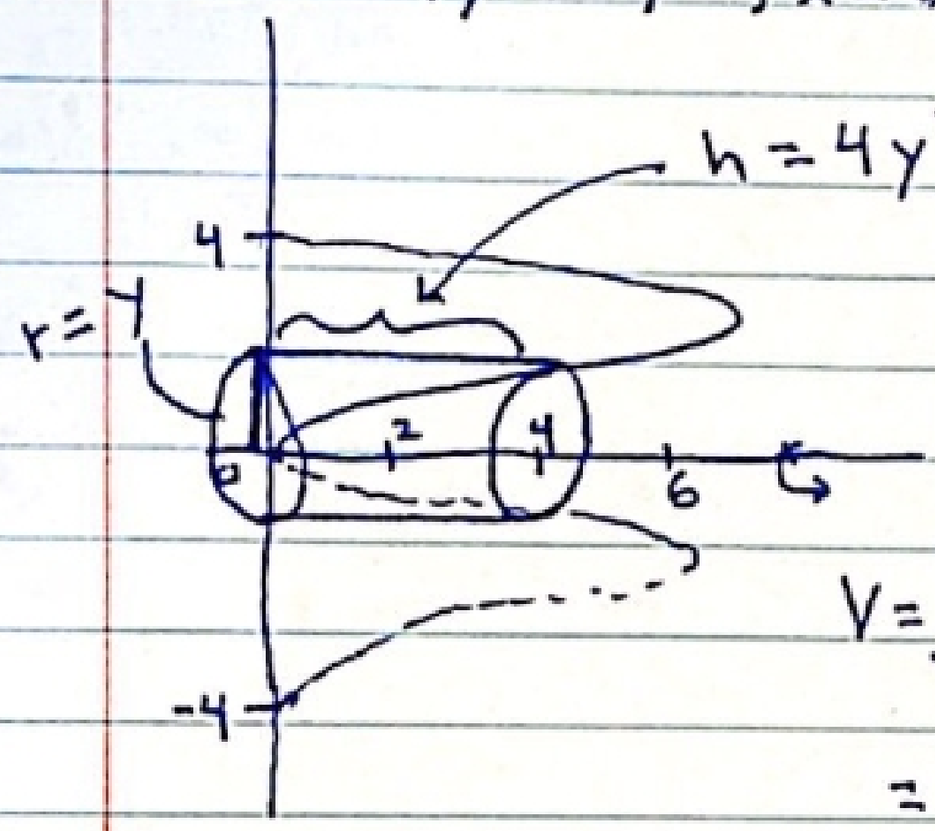
integrate w.r.t x

$h = x^2 - 0 = x^2$

$$V = \int_a^b 2\pi r h \, dx = \int_0^1 2\pi (x)(x^2) \, dx = 2\pi \int_0^1 x^3 dx$$

$$= \frac{2\pi}{4} x^4 \Big]_0^1 = \boxed{\frac{\pi}{2}}$$

2) $x = 4y^2 - y^3$, $x = 0$, about x-axis



Integrate w.r.t y

$$V = \int_0^4 2\pi y(4y^2 - y^3)\,dy$$

$$= 2\pi \int_0^4 (4y^3 - y^4)\,dy$$

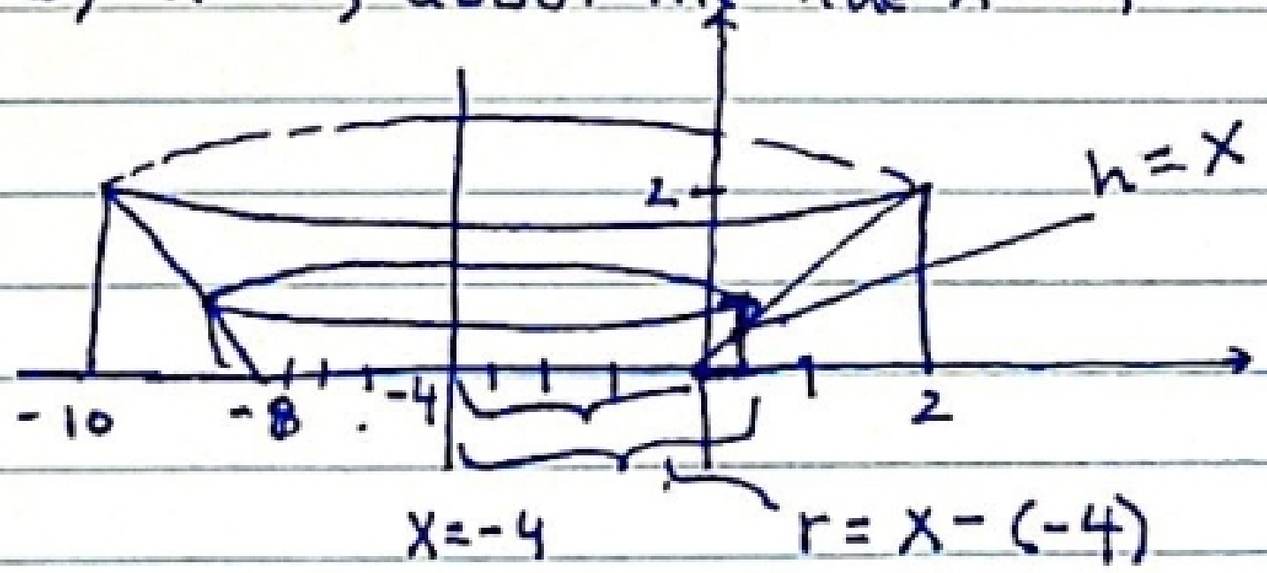
$$= 2\pi \left[y^4 - \frac{1}{5}y^5 \right]_0^4$$

$$= 2\pi \left(256 - \frac{1024}{5} - 0 \right)$$

$$= \frac{512\pi}{5}$$

3) Find the volume using Shell method

$y = x$, $y = 0$, $x = 2$, about the line $x = -4$



$$= x + 4$$

$$V = \int_0^2 2\pi (x+4)x\,dx$$

$$= 2\pi \int_0^2 (x^2 + 4x)\,dx$$

$$= 2\pi \left[\frac{1}{3}x^3 + 2x^2 \right]_0^2 = 2\pi \left(\frac{8}{3} + 8 - 0 \right) = 2\pi \left(\frac{32}{3} \right)$$