Not garenteed a max or min.

10/2/14

local extrema (max/min) in open region.

1) Find critical points.

$\nabla f(P) = 0$ or undefined

2) Analyze critical points

2^{nd} derivative test.

Today: Max/mins on compact region (global)

Region is compact if it is:

1) finite in extent

2) "closed" (if you include boundary.)

If f is continuous on a compact region Ω then it has a global max and a global min in Ω

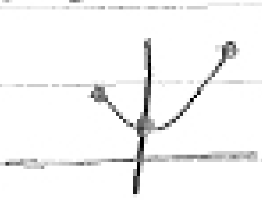
Our Plan:

1) Find Interesting points (critical & end points)

2) Analyze interesting points (plug into "objective" function

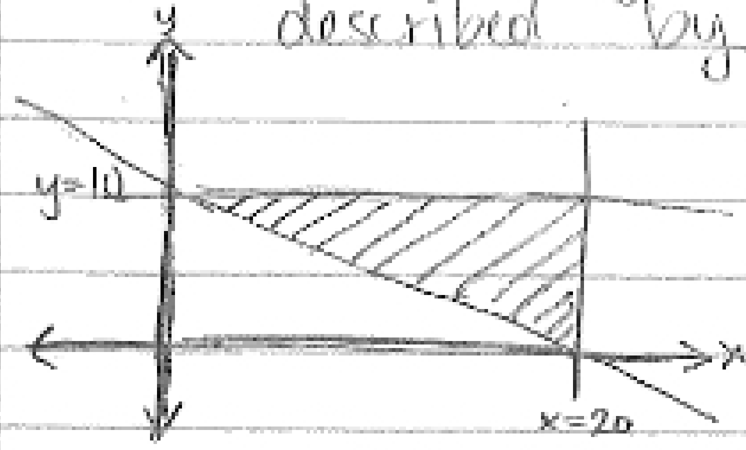
$f(x) = x^2 + 2$

$\Omega = [-1, 5]$ (its closed so its compact)



EX) Find global max & min & where they occur of the objective function

$f(x,y) = x^2 + y^2$ on compact region described by $x \le 20$ $y \le 10$ $x + 2y \ge 20$



* Each boundary needs checked as well as corners & middle.

1)

	Interesting points (p)	f(p)	
interior:	~~(0,0)~~ * Not in region	—	
edge 1:	(20,0)	400	
edge 2:	(0,10)	100	
edge 3:	(4,8)	80	⇒ MIN
corner 1:	(already on		
corner 2:	listed above)		
corner 3:	(20,10)	500	⇒ MAX

Interesting pts.

$\nabla f(p) = 0$ or undefined

$\nabla f(xy) = \langle 2x, 2y \rangle$

$2x = 0$

$2y = 0$ $(0,0)$

edge/boundary 1:

$x = 20$ $0 \le y \le 10$

$g(y) = f(20,y) = 400 + y^2$

$g'(y) = 2y$

set $2y = 0$

$y = 0$ $(20,0)$

Edge/boundary 2:

$y=10 \quad 0 \le x \le 20$

$h(x) = f(x,10)$

$h(x) = x^2+100$

$h'(x) = 2x$

set $2x=0$, $x=0$ $\quad (0,10)$

Edge/boundary 3:

$x+2y=20$

$k(y) = f(20-2y, y)$

$= 400-80y+4y^2+y^2$

$= 400-80y+5y^2$

set $k'(y) = 0$

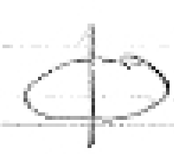
$0 = -80+10y$

$80 = 10y$ $\quad (4,8)$

$8 = y$

f has a global max of 500 at $(20,10)$ and a global min of 80 at $(4,8)$

Find global max & min of objective function $f(x,y) = x^2y$ subject to the constraint $x^2+2y^2=6$ ← ellipse



$x^2 = 6-2y^2 \quad x = \pm\sqrt{6-2y^2}$

whatever

$g(y) = f(\quad , y)$

$= (6-2y^2)y$

$= 6y-2y^3$

$g'(y) = 6-6y^2$

$0 = 6-6y^2$

$y^2=1 \quad \boxed{y=\pm 1}$

when $y=1$

$x = \sqrt{6-2} = \boxed{\pm 2}$

when $y=-1$

$x = \sqrt{6-2} = \boxed{\pm 2}$