DAY 27: THE FOUR ARISTOTELIAN FORMS (9.5); TRANSLATING COMPLEX NOUN PHRASES (9.6); QUANTIFIERS AND FUNCTION SYMBOLS (9.7)

Assigned reading (sections 9.5 and 9.6)

POWERPOINT SLIDE #1

The four Aristotelian forms and how we translate them in modern symbolic logic:

All P's are Q's — $\forall x\ (P(x) \rightarrow Q(x))$ — *Note the* *__conditional__*.

Some P's are Q's — $\exists x\ (P(x) \land Q(x))$ — *Note the* *__conjunction__*.

Note that this does **NOT** *equal* $\exists x\ (P(x) \rightarrow Q(x))$

No P's are Q's — $\forall x\ (P(x) \rightarrow \neg Q(x))$

$\Leftrightarrow\ \neg\exists x\ (Px) \land Q(x))$

Some P's are not Q's — $\exists x\ (P(x) \land \neg Q(x))$

POWERPOINT SLIDE #2

Two tips in the "Remember" box on p. 247:

1) Translations of complex quantified noun phrases frequently employ **conjunctions** of atomic predicates. Examples:

   "A small, happy dog is at home."
   $\exists x[(Small(x) \land Happy(x) \land Dog(x)) \land Home(x)]$

   "Every small dog that is at home is happy."
   $\forall x\ [(Small(x) \land Dog(x) \land Home(x)) \rightarrow Happy(x)]$

POWERPOINT SLIDE #3

2) The order of an English sentence doesn't always correspond to the order of its FOL translation. Examples:

"Max owns a small, happy dog."

∃x[(Small(x) ∧ Happy(x) ∧ Dog(x)) ∧ Owns(max, x)]

Note that the complex noun phrase still comes first in FOL.

"Max owns every small, happy dog."

∀x [(Small(x) ∧ Happy(x) ∧ Dog(x)) → Owns(max,x))

POWERPOINT SLIDE #4

POTENTIAL PROBLEM CASES:

∀x (P(x) → Q(x)) in *vacuously true* cases (i.e., in worlds where the antecedent is **False**, thus it is impossible to find a counterexample)

e.g., **∀x (Tet(x) → Small(x))** ... vacuously **true** in a world with no tetrahedra

Compare "Every freshman who took the class got an A" *if no freshmen took the class!*

(conversational implicature)

e.g., **∀x (Tet(x) → Cube(x))** ... *inherently* vacuous: **can only be true** when asserted of a world with no tetrahedra

POWERPOINT SLIDE #5

Another potentially problematic case:

"Some P's are Q's" does NOT contradict **"All P's are Q's"** ... why not?

(In English, when we use 'some' this usually implies [though it doesn't rigorously entail] that we include the meaning '*not all*'. Our FOL does not include implicatures of this sort, so the existential quantifier strictly means 'some' in the sense of 'one or more' without any upper limit . . . and this can include even 'all' of the objects in question.)

Assigned reading pp.253-255 (9.7)

Refer to **POWERPOINT SLIDES #6 and #7**

(I have posted an answer key on Blackboard for exercises 9.23 and 9.24)