Quiz 2 Review Notes

Math 2280

Spring 2008

Disclaimer - These notes are designed to be just that, notes, for a review of what we've studied since the first exam. They are certainly not exhaustive, but will provide an outline of the material that you will need to know for the second exam. They are intended as a study aid, but they should definitely not be the only thing you study. I also cannot guarantee they are free from typos, either in the exposition or in the equations. Please refer to the appropriate equations in the textbook while you're studying to make sure the equations in these notes are free of typos, and if you find a typo please contact me and let me know. If there are any sections upon which you're unclear or think you need more information, please read the appropriate section of the book or you lectures notes. If after doing this it's still unclear please ask me, either at my office, via email, or at the review session. If you think you need further practice in any area I'd recommend working the recommended problems in the textbook from the sections that cover the material on which you're having trouble.

1 Mechanical Systems Modeled by Linear Equations of Higher Order

We finished off chapter 3 by studying how higher order linear ODEs are applied to mechanical and electrical systems in the real world.

1.1 The Mass-Spring-Dashpot System

The mechanical system that we studied the most was the mass-spring-dashpot system pictured below:

Mass-Spring-Dashpot System

with mass m, damping constant c, and spring constant k. The ODE for this system is:

$$mx''(t) + cx'(t) + kx(t) = 0$$

where $x(t)$ is the mass's displacement from its equilibrium position. Now, if there is an external driving force $F(t)$ (pictured schematically in the picture as the arrow) then we get the potentially more complicated ODE:

$$mx''(t) + cx'(t) + kx(t) = F(t).$$

Now, the most commonly studied situation here is when the driving force is a periodic sinusoidal function $F(t) = F_0 \cos(\omega t - \alpha)$, where F_0 is the amplitude, ω is the angular fequency, and α is the phase, which determines the initial positon. If this is the case then our methods for calculating particular solutions, namely the method of undetermined coefficients, works to solve problems of this type.

If the damping constant c is not present, that is $c = 0$, then the mass-spring system oscillates back and forth with an angular frequency ω_0 given by:

$$\omega_0 = \sqrt{\frac{k}{m}}$$

if the driving function $F(t) = F_0 \cos(\omega t - \alpha)$ has the same frequency as the natural frequency of vibration for the mass-spring system, $\omega_0 = \omega$, then we get the phenomenon of resonance. In this case our solution $x(t)$ will have a factor of the form $At \cos(\omega_0 t - \beta)$ which, because of the t term multiplying the cosine, makes the amplitude of the oscillations grow without bound as time increases.

1.2 Electrical Circuits

An electrical circuit of the form below:

Inductor-Resistor-Capacitor System

has a mathematical model that is fundamentally the same as the model used for our mass-spring-dashpot system with a driving force. Namely, we have the ODE:

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = E(t)$$

So, all the mathematical methods and results from our mechanical system carry over into our electrical system. Note here that Q represents the amount of charge on the capacitor. The movement of charge causes current, so $\frac{dQ}{dt} = I$, where I is the current in our circuit. Frequently we work with current, as in practice it's almost always easier to measure, and so if we take derivatives our equation above becomes:

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}I = E'(t).$$