

Nuke 301 Study Problems, First Exam, 2009

Material: Notes through Chapter 5 Section IV.
Lamarsh Chapter 1.
Lewis Chapters 1, 2, and 4

1. You must compute the rate at which neutrons are produced in a set of reactors. You also must compute the rate at which neutrons are lost in the reactor. The only production mechanisms of any consequence are
- 1) neutron-induced fission in the reactor fuel,
 - 2) spontaneous fission in Cf-252 (only in some of the problems).
- The only loss mechanisms of any consequence are
- 1) absorption,
 - 2) leakage.

Compute *each* of the production rates and also the absorption rate for the cases listed below. Be *very precise* with your notation. Make sure someone else could easily follow your logic and calculations. Then answer the questions.

In parts (a)-(d) the cross sections you are given have been appropriately averaged, not only over all nucleus velocities, but also over all neutron energies. This means that a reaction-rate density is one of these macroscopic cross sections times the total scalar flux – you don't have to worry about energy integrals in (a)-(d).

- a) Shoebox-shaped homogeneous reactor:
 $-a/2 < x < a/2$, $-b/2 < y < b/2$, $-c/2 < z < c/2$,
 operating in steady state. You are given

$$\phi(x, y, z) = A + C \cos\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi y}{b}\right),$$

$$\Sigma_t = 5 \text{ cm}^{-1}, \quad \Sigma_a = 0.2 \text{ cm}^{-1}, \quad \Sigma_f = 0.06 \text{ cm}^{-1}, \quad \nu_{\text{fuel}} = 2.5 \text{ n/fission.}$$

where

$$A = 2 \text{ E}9 \text{ n/cm}^2\text{-s}, \quad C = 1 \text{ E}10 \text{ n/cm}^2\text{-s.}$$

In this part you must also calculate the **leakage rate**. (Note that the reactor is operating in *steady state* – this gives you enough information to figure out the leakage rate once you calculate the other rates!)

You are given that the Cf-252 is distributed uniformly throughout the reactor, and that:

$$N_{252} = \text{Cf-252 density} = 1\text{E}18 \text{ atoms/cm}^3,$$

$$\lambda_{f,252} = \text{decay constant for spontaneous fission} = 0.01 \text{ year}^{-1}.$$

$$\nu_{252} = \text{neutrons emitted per spontaneous fission of Cf-252} = 3.8 \text{ n/fission}$$

- b) Another shoebox reactor, same dimensions and material properties as in part (a). The same Cf-252 is distributed with the same density. This time, though, you are told that:

$$\phi(x,y,z) = A,$$

for all x,y,z in the reactor, and there is no leakage at all. If the reactor is operating in steady state, what is the value of the constant A ?

- c) A spherical reactor, one meter in diameter, critical and operating in steady state. Material properties are as in part (a), except that

$$\Sigma_f = 0.09 \text{ cm}^{-1},$$

and there is no Cf. If the average scalar flux in the reactor is:

$$\bar{\phi} = \frac{1}{Vol} \int_{Vol} dV\phi = 1E9 \text{ n/cm}^2\text{-s},$$

then what is the rate per cm^2 at which neutrons are leaking out of the outer surface ($r=50\text{cm}$)?

- d) A commercial pressurized-water reactor. The reactor has 193 fuel assemblies arranged in a roughly circular pattern (if you are looking down on it). Each fuel assembly is $20 \text{ cm} \times 20 \text{ cm} \times 300 \text{ cm}$.

A nuclear engineer divided each assembly into 20 axial segments (each 15 cm tall), and she did some calculations that produced the following results:

$$\phi_{ij} = \frac{1}{V_{ij}} \iiint_{\substack{j\text{th axial region} \\ \text{of } i\text{th assembly}}} dx dy dz \int_0^{\infty} dE \phi(x, y, z, E).$$

That is, she has this volume-averaged flux, ϕ_{ij} , stored away for $i=1,\dots,193$ and $j=1,\dots,20$. She also has very good estimates for the flux-weighted average cross sections in each axial layer of each assembly:

$$\Sigma_{\alpha,ij} = \frac{1}{V_{ij}\phi_{ij}} \iiint_{\substack{j\text{th axial region} \\ \text{of } i\text{th assembly}}} dx dy dz \int_0^{\infty} dE \Sigma_{\alpha}(x, y, z, E) \phi(x, y, z, E),$$

along with the same kind of average for the total and fission cross sections.

For this reactor, write down expressions, in terms of her estimates and known volumes, for the production rate from fission and the loss rate from absorption. (Ignore other production & loss mechanisms.)

- f) A slab reactor, infinite in the y and z directions, thickness a in the x direction. For this reactor, the production and loss rates are infinite, because the volume is infinite. Since that's not very interesting, you should find expressions for the production & absorption rates **per unit y - z area**. That is, consider a $\Delta y \times \Delta z \times a$ chunk of the slab, compute the rates, and divide by $\Delta y \Delta z$. Your expressions should contain the energy-dependent scalar flux (which is a function of position and energy), the energy-dependent cross sections (which you can assume have been averaged over nucleus motion and are constant spatially).

2. Microscopic cross sections depend fundamentally on a certain speed. What speed?
3. What is the physical interpretation of a macroscopic cross section?
4. What is the physical interpretation of the total scalar flux? If you integrate the total scalar flux over a volume and over a time interval, what is the physical interpretation of the answer?
In general, if you integrate the total scalar flux over a surface area, what is the physical significance of the answer?
5. ~~What is the physical interpretation of the net current density? If you integrate the net current density over a volume what is the physical significance of the answer?~~
6. What is the macroscopic total cross section, at a neutron speed of 2200 m/s, in a mixture of 10 grams of He-4 and 10 grams of hydrogen? Data:
 $\sigma_f^{He} = 0.8$ barns, $\sigma_f^H = 38$ barns.
 Assume an ideal gas at standard temperature and pressure.
7. In the middle of a large homogeneous reactor, the scalar flux is roughly independent of position. Suppose in such a reactor you know that the energy-dependent scalar flux is approximately:
 $\phi(E) \approx \phi_0 / E$ for $1 \text{ eV} < E < 1 \text{ MeV}$, and $\phi(E) = 0$ for $E > 1 \text{ MeV}$.
 If the differential scattering cross section in this reactor is

$$\Sigma_s(E_i \rightarrow E_f) = \begin{cases} 0, & E_f > E_i, \\ \Sigma_{s0} / E_i, & E_f \leq E_i, \end{cases}$$
 then what is the rate (per cm^3 per s) at which neutrons scatter from energies above 1 keV to energies below 1 eV in a single scattering event?
8. In a thermal reactor, for every 100 neutrons emitted in fission, 12 escape while slowing down, and 3 escape after slowing down to thermal energies. No neutrons are absorbed while slowing down. The values of η_T and ν_T in the fissile material are 2 and 2.5, respectively.
- If the reactor is critical, what is the thermal utilization?
 - What is the resonance-escape probability (p)?
 - What is the fast-fission factor (ϵ)?
 - How would the answer to part (a) change if the resonance-escape probability were 0.8 and the fast-fission factor were 1.25?
9.
 - Define (in words) the reproduction factor, usually written as η .
 - Define (mathematically) η in terms of cross sections, assuming a homogeneous reactor.
 - What is the absolute minimum value of η that a breeder reactor could have?