Lecture Notes on Number Systems

1. **Question: Which is the bigger number?**

    5 3 2 (ans 5)

    Alternative question: Which is "five"?

    five V cinq 101 (ans none)

    The point: These are all names of the abstract notion of 'fiveness'. We can invent LOTS of other systems to name numbers.

    NUMBER != NUMERAL

2. **Positional (Polynomial) Number Systems**
    Example: decimal: 346 = 3*100 + 4*10 + 6*1

    general: "***radix*** R" or "base R":

    $$\text{Integers} \quad d_n d_{n-1} \ldots d_0 = \sum_{i=0}^{n} d_i \cdot R^i$$

    and

    $$\text{Reals} \; d_n d_{n-1} \ldots d_0 \bullet d_{-1} d_{-2} \ldots d_{-m} = \sum_{i=-m}^{n} d_i \cdot R^i$$

    Radix Point

    - The "d's" are generally 0, 1, ... R
    - Common notation: ddd_R

    Do some examples: ***binary***, ternary, ***octal***, ***hexadecimal***.

3. **Conversion**
    Radix R - to - Decimal: Compute the expansion

    $$n = d_n R^n + \ldots + d_0 R^0$$

Decimal - to - Radix R

$$\frac{n}{R} = d_n R^{n-1} + \ldots + d_1 R^0, \text{ with remainder } d_0$$

Algorithm:

- Divide succesively by R.
- Note remainder at each step. (order of digits is lsb to msb)

Radix R_1 to Radix R_2

R_1 to decimal to R_2

4. Systems of Special Interest

Binary	base 2	Easy for computers
Octal	base 8	Eas<u>ier</u> for people
Hexadecimal	base 16	Eas<u>ier</u> for people

Show conversion between all of these

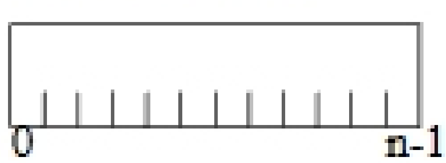
Emphasize hex numbering 0 ... 9ABCDEF

5. Integer Representation in Real Computers

Issues: **Fixed Size, Negative Number Representation**

A. Fixed Size

"The XXX is a 16-bit (vs 32-bit) computer ..."
If "word" size is n bits



there are 2^n possible bit patterns so only 2^n possible distinct numbers

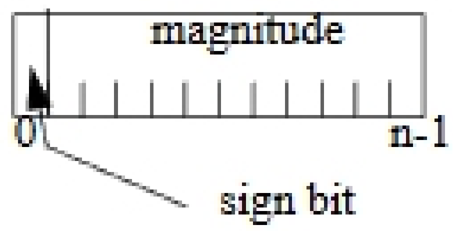
Implications:

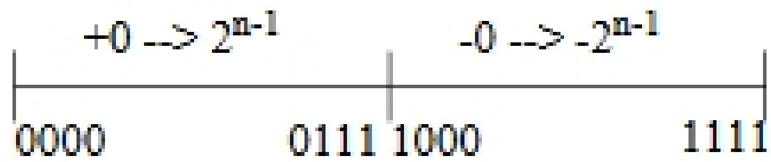
- Cannot represent all possible numbers!!
- Must use some of these bit patterns (half?) to represent negative numbers.

B. Negative Number Representation

i. sign magnitude (the obvious one)

- reserve one bit for the sign
- rest of bits are interpreted directly as the number





Notes:

- Two (2) zeros - generally bad.
- Arithmetic costly (we'll see).

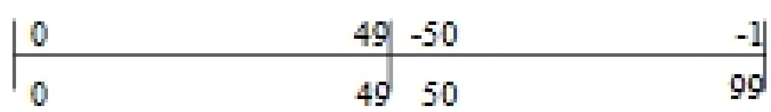
ii. radix and radix-1 complement (also called r's and (r-1)'s)

-k represented as R^n-k

example: base 10 -- 10's complement, n=2.

+3 = +3

-3 = 10^2-3 = 97



Negation is simple: "Complement" every digit and add 1.

100 -k = 99 - k + 1

 99 + 1
-xy

 $\overline{xy}$ + 1

Example (R=10): -(38) = 99 - 38 + 1 = 61 + 1 = 62

-(62) = 99 - 62 + 1 = 38

2's complement (special case):

-k = 2^n -k