

Number Theory

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When talking about division over the integers, we mean division with no remainder.

Definition

Let $a, b \in \mathbb{Z}, a \neq 0$, we say that a divides b if there exists $c \in \mathbb{Z}$ such that $b = ac$. We denote this, $a \mid b$ and $a \nmid b$ when a does not divide b . When $a \mid b$, we say a is a *factor* of b .

Theorem

Let $a, b, c \in \mathbb{Z}$ then

- 1 If $a \mid b$ and $a \mid c$ then $a \mid (b + c)$.
- 2 If $a \mid b$, then $a \mid bc$ for all $c \in \mathbb{Z}$.
- 3 If $a \mid b$ and $b \mid c$, then $a \mid c$.

Corollary

If $a, b, c \in \mathbb{Z}$ such that $a \mid b$ and $a \mid c$ then $a \mid mb + nc$ for $n, m \in \mathbb{Z}$.